Temperature Compensation of a Replica of John Harrison's First Sea Clock

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1 Introduction

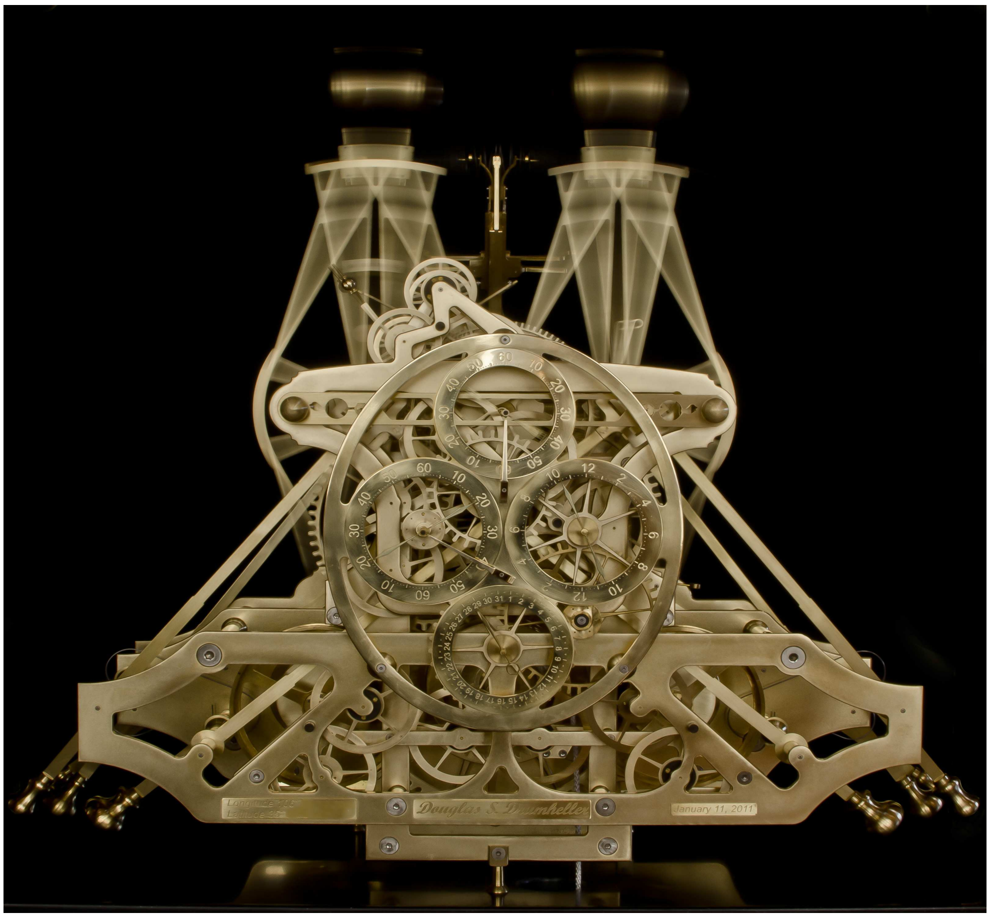

Figure 1: Replica D1. Photo by Dave Cherry. From [1].

As most horologists know John Harrison built four sea clocks during the 18th Century in his effort to win the Longitude Prize. (See Sobel [2]). I first read this story in 1995, and like so many other readers, I was captivated, but perhaps a little more so than many as I decided to build a replica of his Sea Clock H.1. I hope that someday my machine will serve as a navigation instrument to carry longitude across the Atlantic. (See Fig. 1.)

The Sea Clock H.1 was Harrison's first attempt at a navigation instrument and he did make mistakes. In particular he attempted to temperature compensate his machine with what we shall call the *H.1 Compensator*, a complex device which measures the temperature of the surroundings and adjusts the stiffnesses of the balance springs. Unfortunately this compensator makes the clock highly anisochronal [3]; that is, it makes the rate of the clock highly sensitive to the amplitude of the balance swing. Harrison tried to correct the problem by incorporating a remontoire along with a more robust version of this compensator in his second sea clock, H.2. But it didn't work and this ultimately lead to his abandonment of his first two sea clocks followed by nearly 20 years of what must have been a very frustrating effort to develop H.3. Ultimately, of course, he came across the solution in H.4, which got him the prize.

The building of my Sea Clock D1 is described in [1].[1] D1 looks like H.1 but mechanically it is more like H.2 as all of its wheels are brass and it does have a remontoire. It's been going since January 11, 2011 subjected to daily temperature changes often approaching 20 degrees. I've tested it both with and without the temperature compensation provided by the H.1 Compensator. I've found that without the compensator its rate varies with temperature by up to several minutes per day and it becomes the best thermometer in my office. And with the troublesome H.1 Compensator installed the unintended anisochronism leads to similar levels of error. (For a discussion of similar observations in H.1 see Appendix B.) Many people believe that Harrison was a perfectionist who abandoned his first two sea clocks because they exhibited apparently minor rate errors, but in living with my replica over the past years I've come to the conclusion that he was a pragmatist who, facing the obvious flaws in his first design, gave up and moved on.

My situation allows the luxury of a more sentimental view of D1, and I feel that even without H.1's compensator the clock still holds the promise of a functioning navigation instrument. For indeed to determine longitude from it you *merely* need to keep a written log of hourly readings of time and temperature. Then one does the math to come up with a corrected time. I've tried this and it works as the error drops to around 6 sec/day. However, in the absence of a compensator the clock seems incomplete and the real issue that keeps coming to my mind is

> *Could Harrison have designed a better compensator and salvaged H.1 and H.2?*

As we shall see, the answer to this question is, "Yes." With little modification of the mechanism it is possible to install a new compensator that Harrison could have built with the technology and materials of his day. However, he would have had to understand that his original method of altering the spring stiffness with the H.1 Compensator was the source of severe anisochronism and the real culprit in the clock's exaggerated sensitivity to the motion of its ship.

[1] Adopting the name D1 could be very dangerous. First of all the name might already be attached to some other far more deserving object. Second, D1 is really not a replica of H.1 in the strictest sense, but indeed it's close enough that it might be more dangerous not to call it a replica. And third, I've spent more than 20 years building it and I seriously doubt there will ever be a D2.

2 The Replica

Figure 1 is a time-lapse photo of the replica D1, where the semi arcs of the balances are about 6.25 degrees. It shows an implementation of the original H.1 Compensator. Figure 2 is a close-up photo of the compensator pads showing how they extend into the interior of

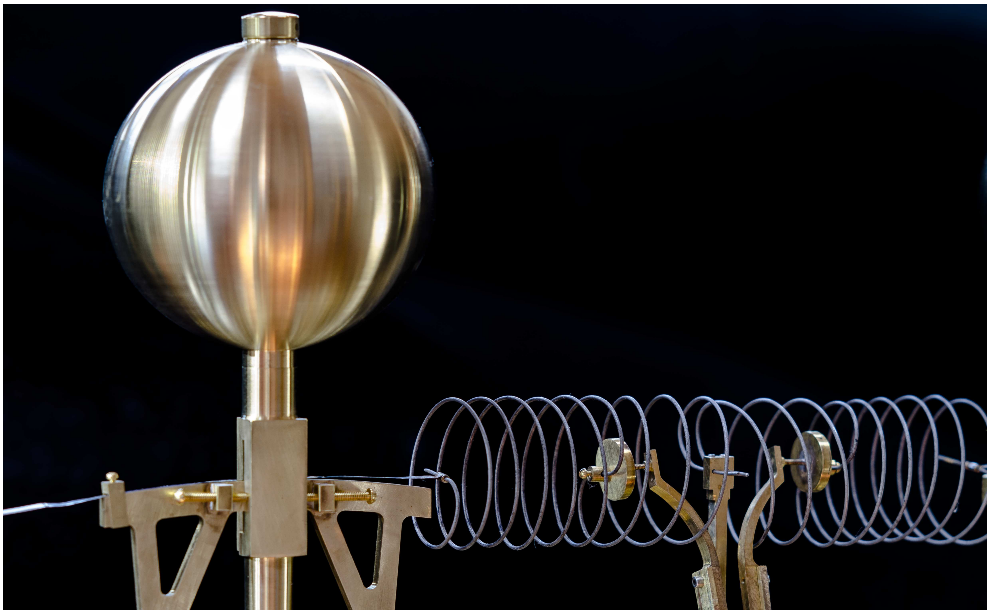

Figure 2: Close-up photo of the balances as viewed from the back of D1. Photo by Dave Cherry. From [1].

the springs to block the contraction of 2.5 coils during a portion of the swing. As a first step in developing a new compensator these pads were removed and discarded leaving only the springs with their thin flexible straps. The straps are attached to the balance frames at the outboard end of each of the four circular cheeks. Thus the springs act along a constant length lever arm to apply the restoring torque to the balance. A temperature increase will cause the balance frame to expand and change the length of the lever arm. But as the balance mass also moves away from the center of rotation, we shall see that the thermal expansion of the frame does not influence the beat rate. However, the beat rate does slow because the stiffnesses of the balance springs decrease with increasing temperature. Understanding this is key to understanding the new compensator proposed in the next section.

3 The D1 Compensators

Figure 3 is a front view of the clock with the motion works and dials removed. Harrison's H.1 Compensator is also absent. Only the Mast A anchoring the stationary ends of the four balance springs remains. We see the two dumbbell balances tied together at Point B with two crisscrossing flexible straps that force the balances to rotate in opposite directions.

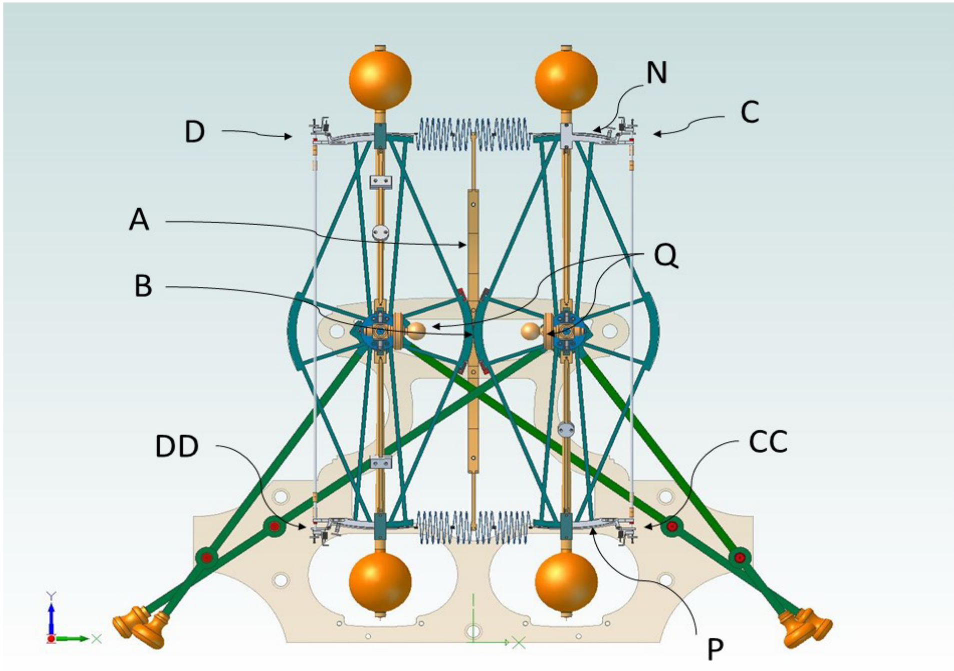

Figure 3: The balances viewed from the front of D1.

Thus when the two top balance springs stretch the two bottom springs contract. Four new *D1 Compensators* labeled C, CC, D and DD are mounted to the balance frames by simply replacing four small retainer plates on the balance arms. To offset the imbalance produced by the addition of these compensators, two counterweights Q are also attached to the arbors using the two keyhole slots in the original balance arbors of H.1. Each of the D1 Compensators has a long glass rod that extends nearly the entire length of the balance. It is the differential thermal expansion of the glass rod with respect to the brass frame that activates its compensator. Detail of Compensator C is illustrated in Fig. 4. Steel pins are glued to both ends of the glass rods using hollow brass collars. The Rod E is rigidly attached to Slider Plate G by means of a set screw (not shown). In the same way Rod F is attached to a corresponding location on Compensator CC at the bottom of this balance frame.[2] While the bottom end of Rod F is fixed to the balance frame the top end can freely slide though Bushing I, made of lignum vitae. Similarly Rod E slides through the bushing on Compensator CC. Plate H is attached to the free end of Rod F. Thus the distance between Slider Plate G and Plate H changes with temperature. By means of Screw L this motion tilts the Lever J and alters the torque that is applied by the Spring M. Slider Plate G can be repositioned on the balance frame to adjust the amount of movement of the Lever J and fine tune the gain of the compensator as required, and indeed Screw L is adjusted so that J never touches the balance frame over the entire operating temperature range. The Spring K holds the Lever J against Screw L while also removing much of the backlash from the

[2]Notice the holes for additional rods, which are an artifact of the development of the D1 Compensators. Two rods per compensator were originally thought to be necessary to provide additional stiffness. Bad idea!

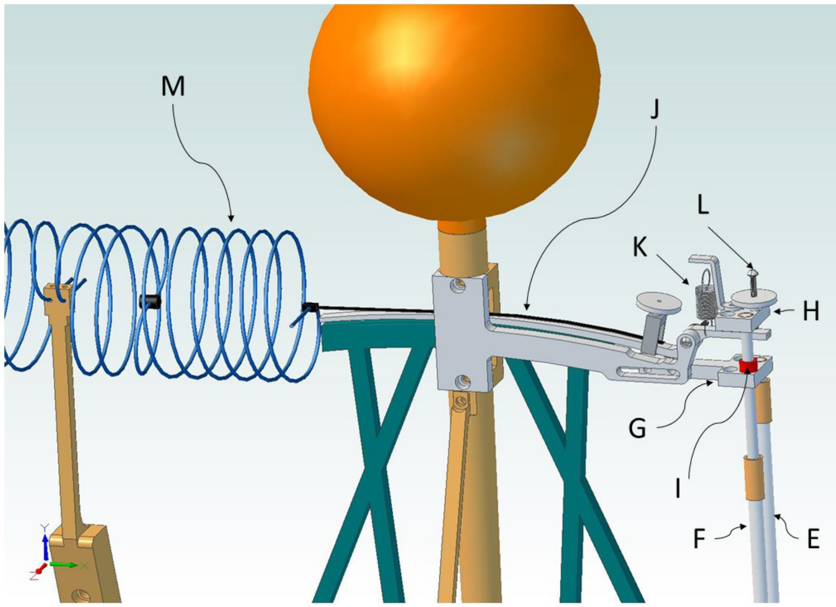

Figure 4: Detail of Compensator C.

linkage. (The spring is a graphic from the McMaster Carr Catalog. Unlike the real spring it cannot be graphically stretched to hook onto the mounting arm above it.)

As you can now see a rise in temperature will cause the brass balance frame to expand more rapidly than the glass rods. This will cause G to move closer to H and tilt the Lever J. There are of course four such levers, which act simultaneously to increase the torque arm of each of the four balance springs and offset their decrease in stiffness.

4 Analysis

Consider the clock with the compensator removed. Its beat time is $\mathcal{T}$. To better focus on the effects of temperature changes we shall ignore changes in $\mathcal{T}$ due to the action of the escapement. The beat changes by an amount $\delta\mathcal{T}$ as the temperature T changes by δT. We define the term $G \equiv -\delta\mathcal{T}/\mathcal{T}$ as the *going* of the clock. When $G = 0$ the clock is unaffected by temperature changes. A positive value means the clock is gaining. In Appendix A we show that

$$G = \left(\tfrac{1}{2}\gamma + \alpha_r - \alpha_{\mathcal{R}}\right)\delta T, \tag{1}$$

where γ is the thermoelastic coefficient of the spring material, α_r is the coefficient of thermal expansion of the lever arm r on which the balance spring acts, and $\alpha_{\mathcal{R}}$ is the coefficient of thermal expansion of the radius of gyration $\mathcal{R}$ of the balance. Without any compensator in place the parameters r and $\mathcal{R}$ are determined solely by the physical dimensions of the brass frame. Therefore both α_r and $\alpha_{\mathcal{R}}$ have values equal to each other and to the thermal expansion coefficient of brass and as such we find that $G = \frac{1}{2}\gamma\delta T$. Thus we see that without the compensator, the going is solely influenced by the spring stiffness and not the expansion of the balance frame.[3]

In situations where γ is not zero, we must provide a compensation mechanism. Harrison tried to achieve this by altering the stiffnesses of the balance springs with the H.1 Compensator, but we choose another option. We shall leave the inertial mass rigidly attached to the balance frame so that $\alpha_{\mathcal{R}} = \alpha_b$, but through the use of the D1 Compensators the lengths of each of the lever arms for the balance springs will now be composed of two parts,

$$r = r_f + \epsilon, \tag{2}$$

where r_f is the position of the compensator on the balance frame and ϵ is the position of the Lever J. (See Fig. 5.) Then from the definition of α_r, given by Eq. 26, and the fact that ϵ is

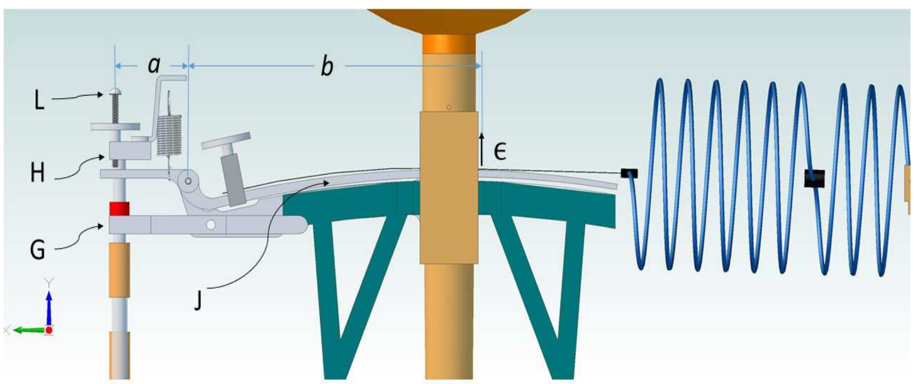

Figure 5: Compensator C viewed from the back of D1.

much smaller than r_f we obtain

$$\alpha_r = \alpha_b + \frac{1}{r}\frac{\delta\epsilon}{\delta T}. \tag{3}$$

Equation 1 now becomes

$$G = \left(\tfrac{1}{2}\gamma + \frac{1}{r}\frac{\delta\epsilon}{\delta T}\right)\delta T, \tag{4}$$

so that the clock will be temperature compensated when the expression contained in the parentheses equals zero: that is, when

$$\delta\epsilon = -\tfrac{1}{2} r\gamma\delta T. \tag{5}$$

This is the motion that each of the Levers J of the D1 Compensators *must* produce to achieve compensation.

[3]Here we see that fabricating the spring from an alloy of elinvar with $\gamma = 0$ will temperature compensate this clock. This method has been adopted by Buchanan of Chelmsford for the Astronomical Skeleton Clock commissioned by Mark Frank, [4].

Next we determine the motion $\delta\epsilon$ that the Lever J *does* produce during the temperature change δT. First notice that the motion of the Plate H with respect to Slider G is equal to the differential thermal expansion of the glass rod with respect the brass frame. As the rod length is $2r$ this differential expansion is $2r(\alpha_g - \alpha_b)\delta T$ where α_g and α_b are the thermal expansion coefficients of the glass rod and the brass frame. This motion is magnified by the lever ratio b/a. Thus

$$\delta\epsilon = \frac{2rb}{a}(\alpha_b - \alpha_g)\delta T. \tag{6}$$

By equating the previous two expressions for $\delta\epsilon$ we see that compensation is achieved when

$$\frac{b}{a} = \frac{\gamma}{4(\alpha_g - \alpha_b)}. \tag{7}$$

Calculating the lever ratio The thermal expansion coefficients of the brass frame and the glass rods are[4]

$$\alpha_g = 2.72 \times 10^{-6} \ /^\circ\mathrm{F} \tag{8}$$
$$\alpha_b = 11.3 \times 10^{-6} \ /^\circ\mathrm{F}. \tag{9}$$

The thermoelastic coefficient of the steel balance springs is

$$\gamma = -169 \times 10^{-6} \ /^\circ\mathrm{F}. \tag{10}$$

Substitution of these values into Eq. 7 yields

$$b/a = 4.92. \tag{11}$$

In comparison the H.1 Compensator uses a compound lever system with amplification of more than 50 between the gridirons and the balance-spring pads, [5].

The D1 Compensators allow for adjustment of both a and b. Sliding G horizontally will change a and adjusting Screw L to raise and lower the Lever J will change b. As illustrated in Fig. 5, the position of b is slightly skewed to the center of the clock because the Mast A is now too short to allow for horizontal alignment of the balance springs.

[4]The value of thermal expansion for the rod is a combination of the values for glass and steel used in the actual implementation. As 2 inches of steel was combined with 14 inches of glass the value was calculated as $(2 \times 6.39 + 14 \times 2.2)/16 \times 10^{-6}$ /°F, where the thermal expansion coefficients of steel and glass are 6.39×10^{-6} /°F and 2.2×10^{-6} /°F, respectively. Now you might imagine that the path leading to this selection of rod materials and lengths was not straightforward. Indeed at one point 1/8-inch diameter birch dowels were tested. They yielded a surprisingly large compensation where the clock gained about 5 sec/day/°F instead of losing 7.3 sec/day/°F. Theory suggests that this is possible only if the thermal expansion coefficient of birch is negative. This seems counterintuitive as we expect heating to expand most materials, particularly their volumes. However, birch is a highly orthotropic material so contraction along the grain could be accompanied by expansion across the grain. A net increase of volume might still occur even as the rod is contracting along its length. But published values of this coefficient are positive; albeit, small. Perhaps the issue is that the dowel diameters were much smaller than the test samples used to obtain the published data. Indeed I only observed one or two growth rings across the diameter of the dowel. As this is a clock project and not a materials-testing project I did not pursue this matter further.

The test results reported in this work were obtained with $a = 0.53$ inches and $b = 2.61$ inches, which corresponds to $G = 0$. This places b at 0.65 inches beyond the centerline of the balance.

Unlike the original frame cheeks which provide a path with a constant radius r, the Lever J only results in a constant radius path at one special position. Thus in general the radius of the path and hence the length of the torque arm changes during oscillation; for example, shortening on the in swing and lengthening on the out swing. This error tends to average out as half of the four compensation levers are always swinging in while the others are swinging out.

5 Testing

The replica is housed in a glass case placed in a room heated with electrical baseboard heaters that are controlled by a commercial solid-state household thermostat. The distance from the replica to the nearest heater is 2 feet while the distance to the thermostat is 15 feet. A temperature gage with a resolution of 1 °F is placed inside the case. The interior temperature of the case appears to fluctuate about the same amount during prolonged periods at a constant thermostat setting.

An 1/8-inch wide black flag is attached to a thin post mounted horizontally on the balance frame at its centerline just below the bottom cheek. The post and flag extend aft towards the rear plate of the clock where a laser photo-gate sensor is located. A small black housing surrounds the sensor. This housing has a narrow slit to allow access for the oscillating post and flag. The clock is shielded from direct sunlight. A Microset Event Timer is used to monitor the photo gate as well as the temperature gage. The duration of the swing of the starboard balance is recorded. The beat time $\mathcal{T}$ is obtained by averaging 30 interruption intervals as the flag crosses the photo gate. This interval corresponds to every two activations of the remontoire.

Anisochronism The semi arc of the balance motion is monitored by measuring the time that the photo-gate beam is blocked by the flag. Smaller times indicate a higher balance velocity as the flag passes across the beam. The amplitude of the balance is determined by dividing a simple scaling factor by the blocking time. Values of going and semi arc were measured for ten hours while holding the room at constant temperature. Each measurement of going G is plotted against its corresponding semi arc with an individual + symbol in Figure 6. The straight dashed line is a least-squares fit of this data. The slope of this line, a measure of isochronism, is 5.34 sec/day/mrad. For comparison in [6] and [7] we reported 5 sec/day/mrad for the isochronism of the replica without a compensator and 50 sec/day/mrad for the isochronism with the H.1 Compensator. Therefore, we conclude the D1 Compensators have little affect on isochronism.

Rapid heating Next we compare two separate 24-hour test measurements of the going G and case temperature T. (See Fig. 7.) The tests without a compensator and with the D1

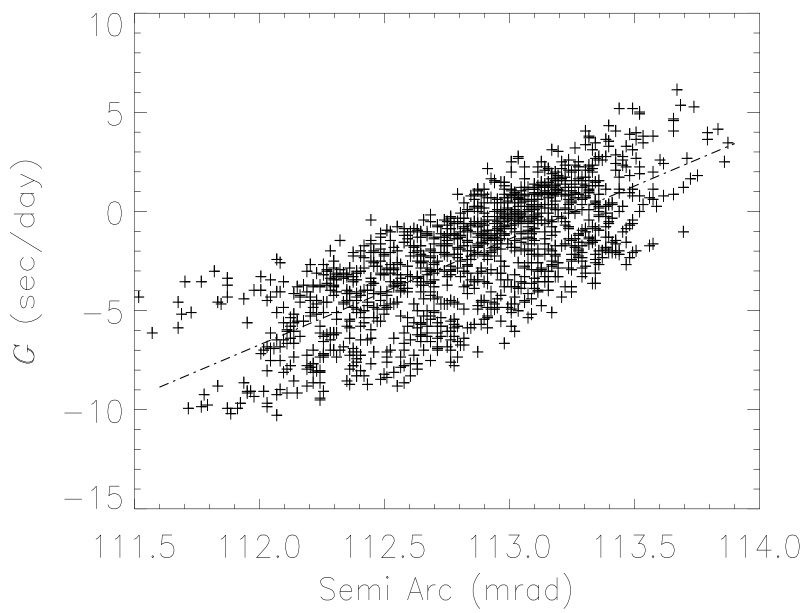

Figure 6: Going versus semi arc with the D1 Compensators.

Compensators are displayed on the left and right, respectively. For convenience we shall call them the uncompensated and compensated tests. The temperature cycles about 15 °F in both tests. The staircase patterns of the dashed curves are due to the coarse resolution of the temperature sensor. In the uncompensated test the going builds to a gain of about 2 minutes a day as the temperature drops, while the going in the compensated test remains relatively constant. It's most significant rate departure occurs as the room is rapidly reheated. Also, although not shown, the semi arc simultaneously dips about 1 mrad at this point. During this period the temperature of each balance spring changes more rapidly than the temperature of the glass compensator rod.[5] Thus the drop in the spring stiffness out runs the motion of Lever J. This effect is also present during cooling but it is far less evident as the room cools far more slowly over a 12-hr period.

A better understanding of the effects of heating and cooling rates on the replica is obtained from Fig. 8, where G is replotted against T for each test. The two tests have been separated by adding 20 sec/day to the going of the uncompensated test. Data are plotted as unconnected points to accentuate the coarse resolution of the temperature sensor, and they are averaged with a box-car filter acting across 30 samples. In both tests we see a hysteresis loop. The upper leg of each loop corresponds to the slow-cooling portion of that test. The lower leg of each loop corresponds to the rapid-heating portion. The loop in the uncompensated test is caused by the slow response time of the temperature sensor. However, the response of the temperature sensor cannot cause the loop in the compensated test. It is suspected that that loop is due to the disparity between the heating response times of

[5]The spring wire cools more rapidly because its diameter is one quarter that of the compensator rod. Moreover, the difference in cooling rates is further exaggerated because the thermal diffusivity of steel is 35 times greater than that of glass.

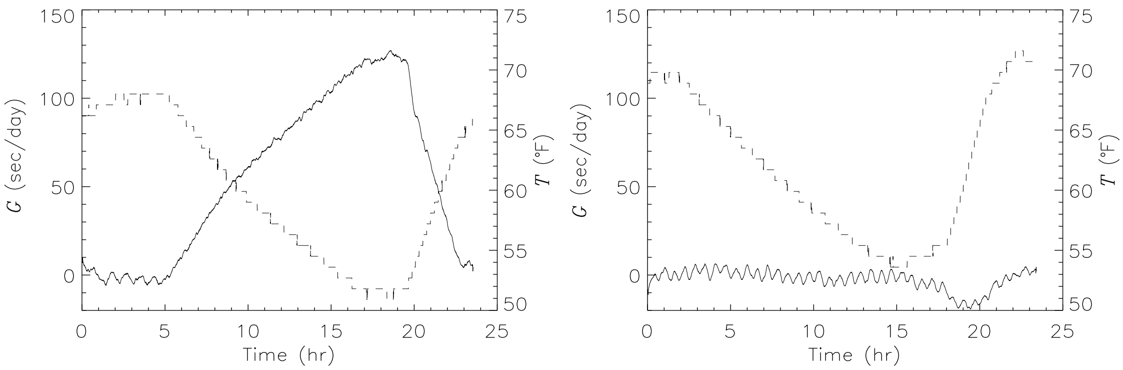

Figure 7: Histories of going (solid) and temperature (dashed) without the compensator (left) and with the D1 Compensators (right).

the spring and compensator rods, the variation in the semi arc, and the backlash in the compensator mechanism.

Slow heating The effect of heating rate is further illustrated by another test with the D1 Compensators. (See Fig. 9.) These data were collected while the room cooled over night and naturally reheated over a period of $9\frac{1}{2}$ hours on the following sunny day. Notice that the temperature range is reduced and the vertical scale has been expanded. We see two sets of data. The lower set were collected and processed as described previously. Notice that the hysteresis loop is now half as deep or roughly 10 sec/day.

Variation of Semi Arc Each data point is now corrected for anisochronism by adding 5.34 sec/day/mrad times the measured change in its semi arc. This produces the upper loop, which is separated by an arbitrary vertical shift of 30 sec/day. Contrasting the two loops in Fig. 9 illustrates the effects of the drop in semi arc during heating. Indeed the upper loop has more tightly grouped values at each temperature. Also the depth of this hysteresis loop is significantly reduced.

Backlash Assuming the remainder of the hysteresis loop is due to backlash, we can estimate the amount of backlash by first writing Eq. 4 as

$$G = \tfrac{1}{2}\gamma\delta T + \frac{\delta\epsilon}{r}. \tag{12}$$

Thus we see that when $\delta T = 0$ the change in rate and the movement of the lever arm are related by

$$\delta\epsilon = rG. \tag{13}$$

As the depth of the hysteresis loop is about G=6 sec/day = 6.9$\times$ 10^{-5} sec/sec and $r = 8$ in, we find that $\delta\epsilon = 0.00056$ in. This suggests the amount of backlash is about one-sixth the

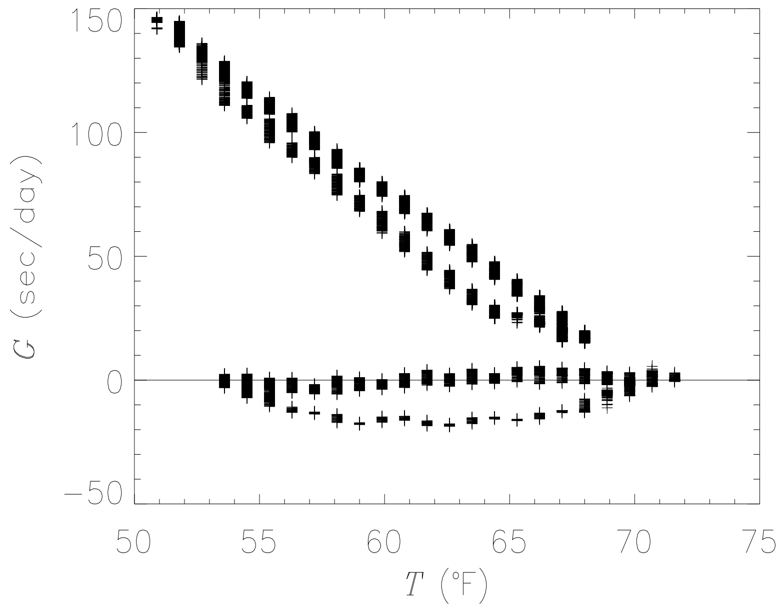

Figure 8: Going versus temperature with and without the D1 Compensators. The latter data are shifted up 20 sec/day.

diameter of a human hair. We suspect that replacing the lever pivot with a flat cantilever spring will reduce the backlash.

Allen Deviation Phillip Woodward [8] gives a simple and very clear description of Allen Deviation, which estimates the fractional instability in the rate of a clock. He is careful to explain that its value is often strongly dependent on the sampling interval employed in the test. Depending on the noise characteristics of the test clock, the instability will either grow or diminish as the length of the sampling interval is increased to yield estimates for longer periods of operation.

The D1 Compensators are new, and so at this point my longest test goes 11 days with the usual daily cycles of slow cooling followed by rapid heating. Gathering data over longer periods is complicated by the fact that the clock must be wound once a day, but there are plans for an electrical rewinding gadget attached to the fusee pull cord. The clock also remains stationary in my office, which is another complication for a device meant to operate on a ship, but that is not going to change until I take her on a sea trial.

With all this in mind, I resampled my 11 days of data over 30 equally spaced intervals. This is equivalent to collecting a rate measurement every 8.8 hours. The Allen Deviation of the resulting sample set is 5.9 sec/day. This roughly duplicates the accuracy I achieved when, as mentioned earlier, I kept a temperature log and did the math to correct the uncompensated clock. We'll see if this number holds up for longer periods of going or even improves. If it does it would have been good enough to receive at least half of the Longitude Prize.

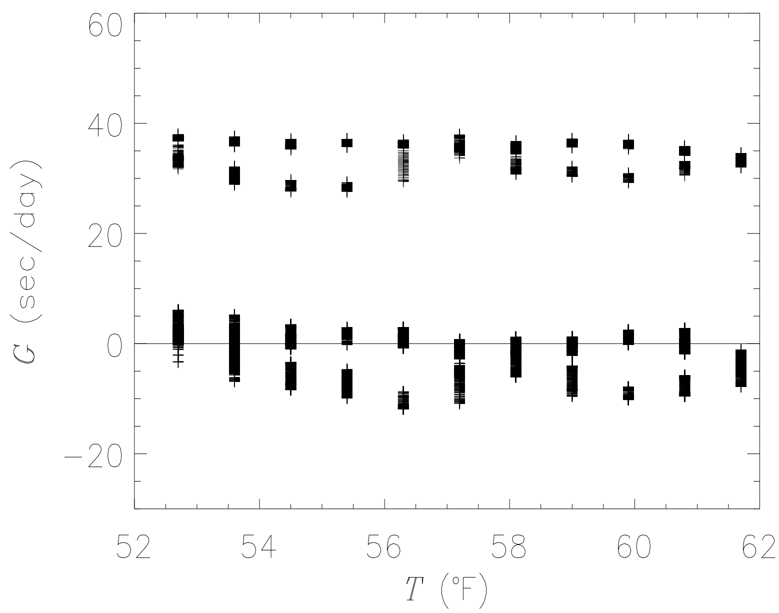

Figure 9: The lower set of data are the going versus temperature with slow reheating. The upper data are the going adjusted for anisochronism. The latter data are shifted up 30 sec/day.

6 Conclusions

It is commonly believed that John Harrison was a perfectionist who abandoned his first two sea clocks solely for minor errors in their going. I however believe he was more pragmatic and saw severe errors due mostly to the large anisochronism induced by his method of temperature compensation. He may not have fully understood this as these rate errors were induced indirectly by the motion of the ship as it first altered the semi arc of the dumbbell balances, which then, acting through anisochronism of 50 sec/day/mrad, produced unacceptable changes in rate. Here we have demonstrated how Harrison could have addressed this problem and perhaps saved his first two sea clocks. Our D1 Compensators are a much simpler design than the H.1 Compensator proposed by Harrison. The design relies on principles and materials that would have been readily available to him.

With help of mathematical analysis we have designed, built, installed and successfully tested the D1 Compensators in our replica. Installation only required the removal of a few screws and small plates from the original clock. Preliminary testing over a period of a few weeks shows that temperature compensation has been achieved with no apparent affect on isochronism. We have employed the differential thermal expansion between the brass balance frame and long glass rods; however, Harrison might have chosen other materials such as wood or steel. He would have quickly understood that the clock itself is an excellent device for determining the coefficient of thermal expansion of any material that he could form into 16-inch long rods.

But indeed as my work has progressed it has become clearer to me that the Sea Clocks

H.1 and H.2 could not have won the Longitude Prize. The anischronism induced by their temperature compensators was just too large. It's what drove Harrison to move on to H.3. But with the help of the D1 Compensator these sea clocks might have stood a chance.

References

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[2] Dava Sobel. *Longitude*. Walker, New York, 1995.

[3] D. S. Drumheller. Anisochronism of John Harrison's first sea clocks, H.1 and H.2. *Horological Science Newsletter*, 3:2–12, 2014.

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A Derivation of Going

Here we shall derive the expression for the change in rate of the uncompensated balance system. This requires the definition of a number of terms describing the geometry of each of the two balances illustrated in Fig. 3. It is sufficient to consider only one of the balances, the port side, to which two balance springs are attached.

This balance has a mass denoted by M. The mass M times the square of the radius of gyration $\mathcal{R}$ of the balance equals the second moment of inertia of the balance. For a simple balance where the mass is concentrated in the two balls and the mass of the frame is negligible, $\mathcal{R}$ is equal to the distance between the center of rotation of the balance and the center of mass of either of the balls. For the replica $\mathcal{R} = 8.6$ in, which is slightly less than the distance between the balls and the center of rotation. The radius of the cheeks N and

P is denoted by r. (See Fig 3.) This is the torque arm on which the spring acts to produce the restoring torque to the balance.

Each helical balance spring is fabricated from steel wire with a shear modulus denoted by μ. The spring rate K is proportional to μ though the expression

$$K = k\mu, \tag{14}$$

where k is a constant determined by the diameter of the wire, the radius of the helical coil and the number of coils in the spring. While the value of the parameter k is sensitive to a change in temperature, that change is significantly smaller than the temperature sensitivity of μ. For our purposes we can, without loss of generality, assume that all of the temperature sensitivity of K resides in μ alone, and k does not change with temperature .

The balance is illustrated in the position known as bottom dead center, BDC. The rotation away from this position is denoted by θ, which is positive for a counterclockwise rotation. During this rotation the bottom spring stretches and the top spring contracts. The deflection of the bottom spring is denoted by Δ where

$$\Delta = r\theta. \tag{15}$$

The force F resulting from this deflection is

$$F = -K\Delta = -k\mu r\theta, \tag{16}$$

and the torque T is

$$T = -k\mu r^2\theta, \tag{17}$$

where the minus sign indicates that the torque opposes the rotation θ. The top spring produces an identical result so that Newton's equation of motion for the balance then becomes

$$-2k\mu r^2\theta = M\mathcal{R}^2\frac{d^2\theta}{dt^2}. \tag{18}$$

Solution of this differential equation shows us that the frequency of harmonic oscillation ω of the balance is

$$\omega^2 = \frac{2k}{M}\mu\left(\frac{r}{\mathcal{R}}\right)^2. \tag{19}$$

The time to complete half an oscillation is called the beat of the balance $\mathcal{T}$. It is related to the frequency by

$$\mathcal{T} = \pi/\omega. \tag{20}$$

Therefore

$$\mathcal{T}^{-2} = \frac{2k}{\pi^2 M}\mu\left(\frac{r}{\mathcal{R}}\right)^2. \tag{21}$$

The temperature is denoted by T. Changing the temperature by an amount δT will result in the changes $\delta\mathcal{T}$, $\delta\mu$. δr and $\delta\mathcal{R}$. To determine how these changes are interconnected we first take the logarithm of the previous equation to obtain

$$-2\ln\mathcal{T} = \ln\frac{2k}{\pi^2 M} + \ln\mu + 2\ln r - 2\ln\mathcal{R}. \tag{22}$$

The differential of this expression yields the required result,

$$-\frac{\delta\mathcal{T}}{\mathcal{T}} = \frac{1}{2}\frac{\delta\mu}{\mu} + \frac{\delta r}{r} - \frac{\delta\mathcal{R}}{\mathcal{R}}. \tag{23}$$

Next we simplify the notation by defining some new terms. The *clock going* is

$$G \equiv -\frac{\delta\mathcal{T}}{\mathcal{T}}. \tag{24}$$

The *thermoelastic coefficient* of the spring material is

$$\gamma \equiv \frac{1}{\mu}\frac{\delta\mu}{\delta T}. \tag{25}$$

The *thermal expansion coefficient* of r is

$$\alpha_r \equiv \frac{1}{r}\frac{\delta r}{\delta T}. \tag{26}$$

The *thermal expansion coefficient* of $\mathcal{R}$ is

$$\alpha_{\mathcal{R}} \equiv \frac{1}{\mathcal{R}}\frac{\delta\mathcal{R}}{\delta T}. \tag{27}$$

Thus Eq. 23 becomes

$$G = \left(\tfrac{1}{2}\gamma + \alpha_r - \alpha_{\mathcal{R}}\right)\delta T. \tag{28}$$

Uncompensated Balance When the compensator is not present, the lengths r and $\mathcal{R}$ represent the linear dimensions of the brass balance frame. Thus $\alpha_r = \alpha_{\mathcal{R}} = \alpha_b$ where α_b is the thermal expansion coefficient of brass, which is commonly listed as $11.3\times10^{-6}/°\text{F}$. Thus the going becomes

$$G = \tfrac{1}{2}\gamma\delta T. \tag{29}$$

Listings of the property γ are not as easily found; however, we have measurements of $G/\delta T = -7.3$ s/day/°F $= -84.5\times10^{-6}/°\text{F}$, for the replica without temperature compensation. Thus the thermoelastic coefficient for the steel balance spring of the replica is $\gamma = -169\times10^{-6}/°\text{F}$. The negative value of course means a rise in temperature weakens the spring.

It is important to remember that by our definition γ is the thermoelastic coefficient of the shear modulus μ of an isotropic solid. The shear modulus is the physical property that gives the helical steel spring its stiffness. However, an isotropic solid has two physical properties that define its stiffness and the other is called Young's Modulus E. The flat mainspring usually found in watches as well as the spiral spring found in H.3 owe their stiffness to Young's Modulus. The thermoelastic coefficient for E, defined as $\frac{1}{E}\frac{\delta E}{\delta T}$, is not necessarily equal to γ.

Compensated Balance Certain nickel-steel alloys known as elinvar have values of γ and also $\frac{1}{E}\frac{\delta E}{\delta T}$ that range across zero, and elinvar alloys with thermoelastic coefficients on the order of one percent of the replica spring are commercially available.[6] The replica could be compensated by fabricating a spring from one of these alloys. Unfortunately elinvar, discovered in 1896, was unavailable to Harrison. Consequently to achieve compensation so that $G = 0$ we shall alter the balance design to adjust α_r so that

$$\alpha_{\mathcal{R}} - \alpha_r = \gamma/2. \tag{30}$$

B Fletcher's Restoration of H.1

In 1932 Rupert Gould, [9] rescued H.1 from oblivion, but he did not restore the H.1 Compensator to functionally. That was done almost twenty years later in 1951 by D. W. Fletcher, [5] and [10]. Fletcher reported that the restored compensator, "...works, and works well." But he qualified that statement with, "The actual performance depends on keeping the arc of swing constant, and this again depends on the mainspring output exactly matching the fusee." Something that he did not do. Remember, H.1 does not have a remontoire. With the compensator installed he, *"... found that by altering the lengths of the wires connecting the balances to the balance springs there is a very considerable alteration to the rate. (A length) alteration of as little as 1/2 mm will make a difference of several minutes a day."* This last statement is perhaps the most interesting. While Fletcher believed it demonstrates a virtue in the compensator design, it actually demonstrates that the H.1 Compensator, when properly adjusted, makes H.1 highly anisochronal. (See Drumheller [3].)

Fletcher also replaced the balance springs in H.1. The original springs are wound first in one direction for 5.5 coils and then in the other direction for 2.5 coils. (See Fig. 2.) The H.1 Compensator acts at the junction between these two segments to block the action of 2.5 coils for a portion of the swing. But as the clock was losing about 50 minutes/day, Fletcher decided to modify the springs so that the 2.5-coil segments have, "... an initial compression so that they (are) kept out of action for a longer period of time." Indeed a simple calculation suggests they would need to be kept out of action *most of the time* to produce this increase in rate. Fletcher's modification would have effectively disconnected the compensator from the clock. After testing this configuration he reports, "...that I have obtained a rate of 8 seconds a day with a variation during the 24 hours of 8 seconds." I interpret this to mean he measured the rate over a single day and probably in a lab while holding temperature constant.

[6] For example, see www.specialmetals.com, but be aware that the differences between the two thermoelastic coefficients are confused on this site. This may well indicate that the measured differences are not all that significant.