PERFECTING THE HARRISON TWIN PIVOT GRASSHOPPER ESCAPEMENT

Perfecting the Harrison Twin Pivot Grasshopper Escapement

First Edition

**This publication is an extract from
'Perfecting the Harrison Grasshopper Escapement',
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published 2009.**

Guidance in basic concepts from an earlier chapter of the parent publication has also been included, in order to create a 'stand-alone' publication.

As much possible, inapplicable references to excluded parts of the parent publication have been removed.

**Figures retain their original numbers
(7.01 to 7.45).**

The term "replica" must not be taken to mean "exact copy".

This publication is not intended to be a thorough or complete constructional guide. Information has been supplied, in good faith, in order to assist construction by those with sufficient skills and experience.

All figures and diagrams are illustrative only.

CONTENTS

INTRODUCING THE TWIN PIVOT GRASSHOPPER ESCAPEMENT

The first grasshopper escapement created by John Harrison was fitted retrospectively to a tower clock created by him for stables at Brocklesby Park, Lincolnshire, due to a failure of his anchor escapement interpretation to operate reliably. Two pallet arms were mounted upon separate pivots, from which the name 'twin pivot grasshopper' has been derived. As far as we can be certain, the Brocklesby escapement was the only twin pivot version of the grasshopper ever made by Harrison, although he may have applied the arrangement to some of his early longcase wooden movement regulators and subsequently substituted single pivot grasshopper escapements.

An inquisitive mind is compelled to understand why Harrison apparently abandoned the twin pivot escapement after such limited use. The Brocklesby grasshopper has delivered efficient, almost continuous service since the day it first came to life, almost three centuries ago. There can therefore be no doubt that it is, at the very least, a reliable escapement, offering excellent longevity. Unfortunately, Harrison offered no clues. He was, without doubt, a scientist and engineer who applied ruthless logic to every one of his actions. If a deficiency in a device or method was exposed, he would either attack the problem until solved, or dispense with it altogether, regardless of the personal consequences or time and effort already invested. A study of his gargantuan efforts to devise a practicable longitude timekeeper provides ample evidence of his fiercely uncompromising attitude. We might conclude, therefore, that there may have been sound reasons for abandoning the twin pivot arrangement.

As will be demonstrated by the analysis presented herein, the twin pivot configuration would appear to offer valuable performance benefits, when compared to the single pivot arrangement. It may well have been that, given a slightly longer life span, Harrison would have identified those invaluable characteristics and adopted it as his preferred solution.

It has been suggested that the eighteenth century process of designing the twin pivot geometry, using graphical methods, presented insurmountable difficulties. However, the task, if thoughtfully approached, is merely tedious and time consuming, rather than impossible. It would, therefore, be doubtful that a man of Harrison's proven tenacity would have been dissuaded by such relatively insignificant challenges.

MECHANICAL ARRANGEMENT

Figure 7.01 (next) shows the escapement, escape wheel and pendulum in illustrative form. Colour is used for clarity of explanation, rather than accurate representation of materials. Component materials are merely sensible or typical suggestions. Two views are incorporated within the single illustration: the uppermost figure is a view from above looking vertically downwards and the lower figure is a view from the front looking horizontally rearwards. The pendulum is a greatly abbreviated, symbolic representation of the physical reality, which would be over a metre in length, dwarfing the escapement. The pendulum is depicted as vertical and at rest and the (brass) escape wheel is bereft of motive power. Notwithstanding the situation in Figure 7.01, driven escape wheels described herein should be assumed to normally rotate clockwise, unless stated otherwise. The escape wheel tooth count of sixty has been chosen entirely because the consequent escapement proportions are ideal for the purposes of clear illustration and explanation; it should not be assumed to suggest optimum configuration or performance. It may be useful to note that the illustrated escape wheel has a tooth tips pitch circle diameter of just over seventy millimetres.

In Figure 7.01, using accepted terminology, the wooden **'entry pallet'** is the first pallet to be met by any chosen escape wheel tooth as it 'enters' the escapement. By the same reasoning, the **'exit pallet'** is the last pallet encountered by any escape wheel tooth as it ‘exits’. Pallet **'nibs'** are formed at the ends of the **'pallet arms'**. The pallet arms are usually weighted at the **'tail'** ends, perhaps with circular inserts of lead, to such a degree that they are tail heavy. The arms are, therefore, constantly inclined to rotate clockwise about their (possibly brass) **‘pallet pivot pin’**, unless constrained to do otherwise. The red and green items are called **'composers'**, usually in brass and described in detail shortly. A pallet nib **'locking corner'** is the internal corner formed by the inverted 'V' at each nib end.

Cantilever mounting of each pallet arm and its paired composer, as illustrated, is a mechanically sound option (if correctly executed), offering potential advantages in terms of installation, manufacture, assembly and maintenance. Not least, the fascinating motions of all components are more easily viewed. It must, however, be bourne in mind that such an arrangement may not be suitable for every installation.

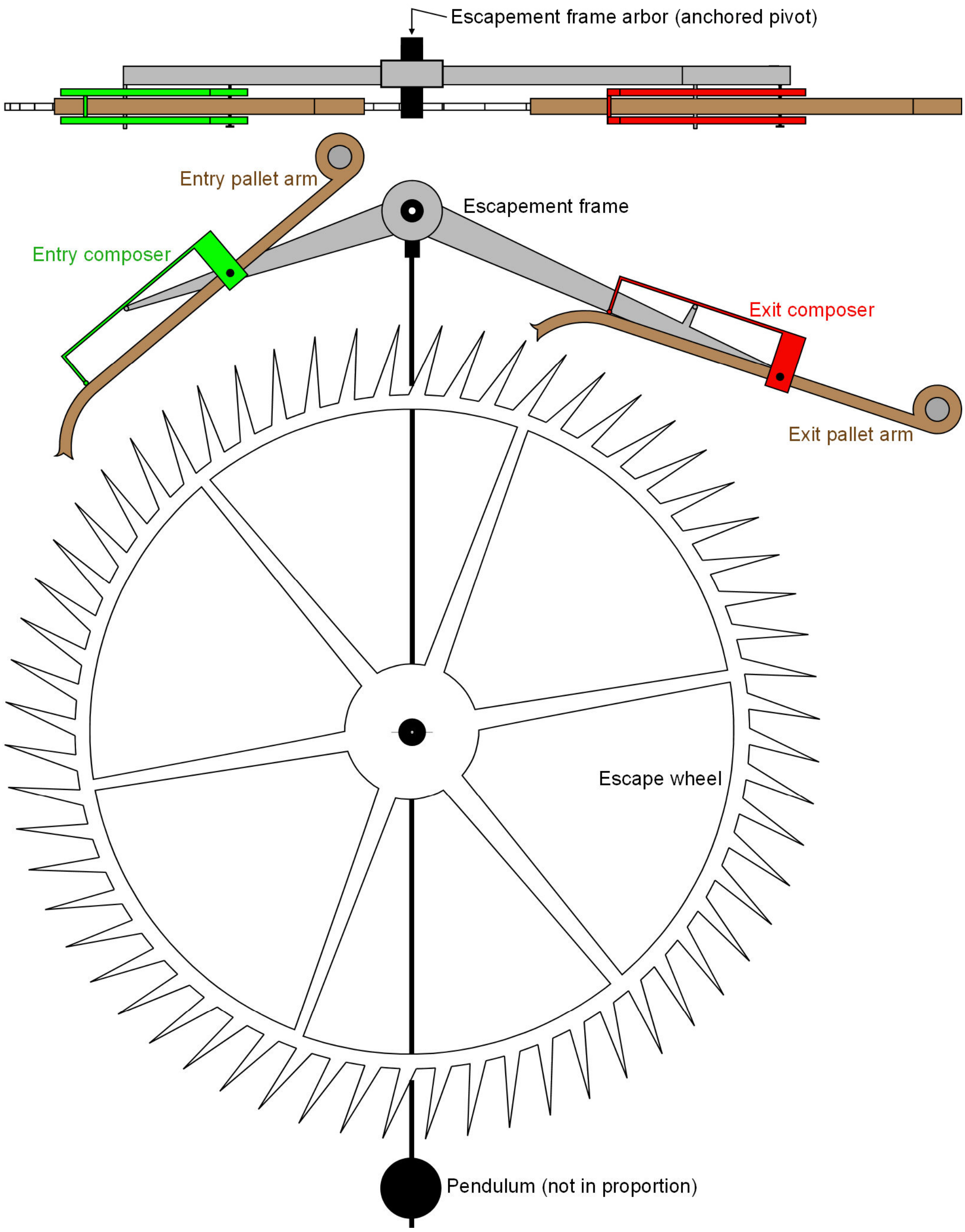
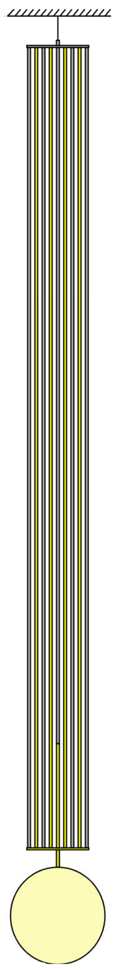


Figure 7.01 - Twin pivot grasshopper escapement mechanical arrangement.

Figure 7.01 illustrates two separate pivots for the pallet arms, affixed rigidly, without freedom to rotate or slide, to the brass or steel escapement frame, which is, in turn, affixed to the pallet arbor, more appropriately referred to as the **'escapement frame arbor'**. A pendulum crutch (not shown, for simplicity), in brass or steel, is affixed to the escapement frame arbor, transmitting torque from the escapement to the pendulum Each end of the escapement frame arbor incorporates **'knife edge pivots'** (again not shown, for simplicity), consisting of small, brass or hardened steel, V-shaped arbor ends rocking in fine, sharp, axial, V-shaped grooves formed on the upper surfaces of flat glass pivot supports. Knife edge pivots are virtually friction free and require no lubrication, those properties being a universal feature of Harrison's devices (longitude 'watches' excepted). For simplicity and to avoid unnecessary clutter, the pendulum is represented by a single broken line, attached without freedom to the escapement frame. In reality, Harrison’s wonderful gridiron pendulum (loosely illustrated below) would be an almost obligatory fitment.

Mounted freely and independently upon each of the pallet pivot pins are additional components, typically of brass, called **'composers'**, one for the entry side (coloured green) and one for the exit side (in red). Both composer are clearly nose heavy, the **'nose'** being the free end. Thus, the entry composer (green) is constantly inclined to rotate anticlockwise about its own pallet pivot pin and the exit composer (red) is also constantly inclined to rotate anti clockwise about its own pallet pivot pin. The interaction between each composer and its paired pallet arm will be described in careful stages in the next section.

Note very carefully the slender, grey, forward extension at the extreme left hand end of the escapement frame, which is somewhat easy to overlook; it prevents downward movement of the entry composer beyond the illustrated position. A similar forward extension, located approximately three quarters of the way down the right hand side of the escapement frame, serves the same purpose for the exit composer. Those extensions will be referred to as the composer 'stops'.



Harrison’s ingenious gridiron pendulum.

Typically a metre or thereabouts in overall physical length. Nine parallel rods (wires), four of steel, four of brass, one (central) of brass and steel joined by a ‘tin whistle’ compensation adjuster. Put simply, adjacent rods are joined rigidly at their ends such that, as atmospheric temperature increases (or decreases) the combined expansion (or contraction) of the brass rods is offset, to the necessary extent and to an adjustable degree, by the expansion (or contraction) of the steel and central brass/steel rods.

COMPONENTS AND THEIR BEHAVIOUR

Independent component motion occurs for nothing more than brief moments during normal escapement operation. There are constant relative motions and frequent interactions, which is why, amongst other things, the grasshopper escapement is so fascinating to observe.

When contact between two components occurs, the more dominant (which will be clearly identified in the course of any future descriptions) will thereafter dictate the motion of the other component. The entire, apparently complex escapement cycle of operation is nothing more involved than a repeating sequence of simple contacts. An unambiguous understanding merely requires the patient observation of every discrete interaction, presented in a logical sequence. That process, painstakingly presented shortly, will no doubt prove tedious for those who are quick to grasp such concepts.

This section presents the twin pivot grasshopper escapement in reducing states of 'undress', beginning with the completely bare escapement frame and rigidly attached pendulum. Components will be added in a logical sequence, accompanied by explanations of the functioning of each. Without apology, there will be some repetition of explanations, in order to guarantee that a crystal clear understanding of the individual and combined behaviour of all components is attained.

CONVENTIONS

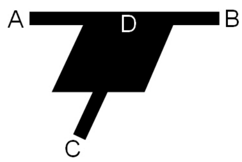
Unless stated otherwise: all views are from the front of an imaginary clock movement, all escape wheels normally rotate clockwise and all dimensions are in millimetres (mm) and degrees (deg).

A key to the symbolic conventions used in the illustrations that follow is as follows:

'Anchored pivot' (greatly enlarged).
Rigidly incorporated into an immovable object. ‘Earthed’.

'Travelling pivot' (greatly enlarged).
Rigidly attached to a moving object. Moves in unison with that object.

'Rigid attachment' (greatly enlarged).
Prevents relative movement between attached objects
A, B and C rigidly connected at D.



Small, symbolic **pushing hands, or a single hand**, (greatly enlarged, right) indicate that a component is being held firmly in the position shown, or pushed in the direction shown, by an imaginary assistant. In order to avoid any possibility of misunderstanding, please note that the symbolic hands are, most emphatically, not to scale.

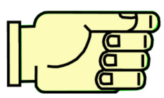


Figure 7.02 (next) represents a view from the front of the twin pivot grasshopper escapement, with many of the components removed. The escapement frame is shown attached rigidly to the escapement frame arbor, which is an anchored pivot (anchored pivots cannot move up, down or sideways, but can rotate freely). The pendulum, or more precisely the upper part of the rod of the pendulum, is shown as a bold, broken line, attached rigidly to the escapement frame, for simplicity of illustration. The pendulum is illustrated as being held in a vertical position by two opposed symbolic hands (not to scale), .

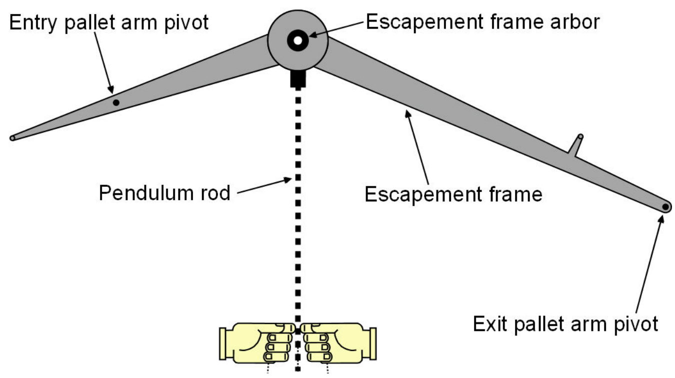


Figure 7.02 - Escapement frame, pendulum held in the vertical position.

Figure 7.03 (next) shows the effect of releasing the hold on the pendulum and applying and maintaining a clockwise torque (twisting effort) about the the escapement frame arbor, represented by the single symbolic hand pushing upwards near the entry pallet arm pivot pin. By virtue of the applied torque, the rigidly connected pendulum and escapement frame are forced to rotate clockwise about the escapement frame arbor. When the applied torque is balanced by the the torque produced by the pendulum and escapement frame, the assembly takes up a stationary position, with the pendulum to the left of vertical. The pendulum is shown, for convenience of illustration, aligned with one of three markers, which are parts of radials from the escapement frame arbor. For future reference, the two outer markers will define the *normal* extent of the pendulum's swing (although it actually swings slightly beyond those positions, of which more later) and the central marker indicates the vertical position of the pendulum.

In normal operation, the escapement delivered torque driving the pendulum need only be sufficient in magnitude to maintain the necessary degree of swing (**'pendulum arc'**), making up for any energy losses due to air resistance and flexing of the suspension spring. It is important to bear in mind that operating torques and forces are, therefore, significantly lower than those suggested by the illustration, by virtue of the pendulum being in motion and already possessed of energy, rather than stationary, as shown here.

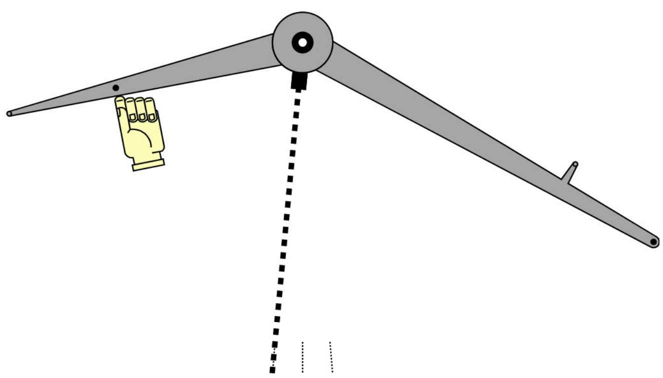


Figure 7.03 - Clockwise torque applied to the escapement frame arbor.

Figure 7.04 (next) mirrors the scenario depicted in Figure 7.03, in that an abnormally high anticlockwise torque has been applied and maintained to the escapement frame arbor. The pendulum and escapement frame rotate anticlockwise about the escapement frame arbor until equilibrium is achieved, at which point they take up the stationary positions shown.

If the sequence of Figures 7.03 to 7.04 was to be repeated in a continuous cycle, it would demonstrate how the motion of the pendulum could be sustained, by alternately applying clockwise and anticlockwise torque to the escapement frame arbor at the required times and in the required amounts.

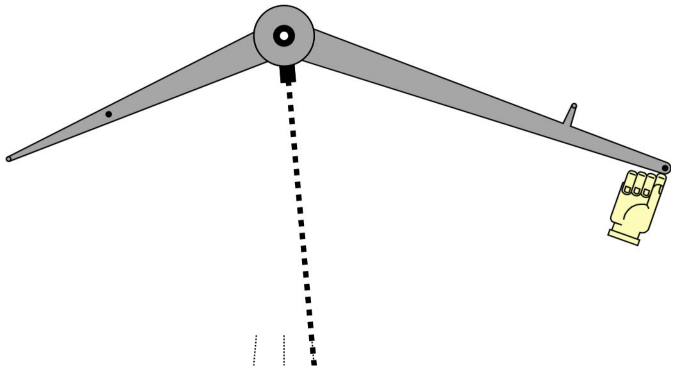


Figure 7.04 - Anticlockwise torque applied to the escapement frame arbor.

Figure 7.05 (next) - The two composers have been added to pallet arm pivot pins of the previous configuration, the entry composer being to the left (in green) and the exit composer to the right (in red). In this illustration, green and red serve to emphasise the unfamiliar forms of the composers. In future illustrations, green and red will only be allocated when an applicable composer moves, relative to the escapement frame, from the **'resting'** positions shown in Figure 7.05. The composers are obviously nose heavy and are at all times attempting to rotate anticlockwise about their respective pallet pivot.

Although the pendulum is shown to be held in a vertical position, this is in no way relevant to the immediate explanation of composer operation.

Two broken circles highlight the points of contact between the resting composers and the small forward extensions to the escapement frame (the composer 'stops'). Any normal movement of the escapement frame and pendulum will not affect the composer resting positions relative to the escapement frame; the entire assembly will simply rotate, otherwise unaffected and unaltered, about the escapement frame arbor.

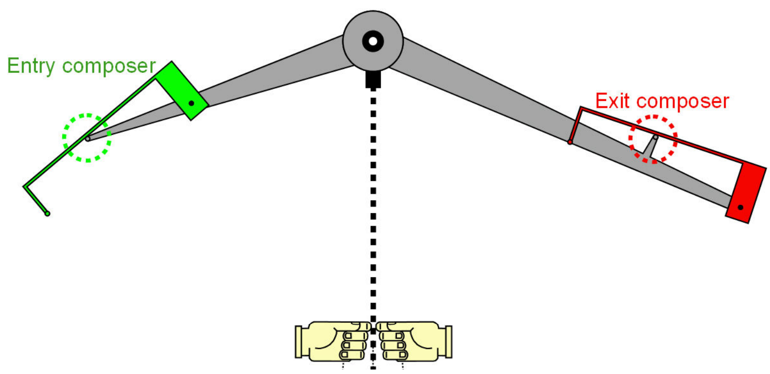


Figure 7.05 - Composers resting upon escapement frame, pendulum held in the vertical position.

Figure 7.06 (next) - The nose of each composer has been raised by an externally applied force, indicated by symbolic pushing hands. In order to provide a clear illustration, the amount of composer movement is considerably greater than that experienced in normal escapement operation. The composers have broken contact with the escapement frame stops (highlighted by the broken circles) and each composer is, therefore, free to generate resistance to the applied force, in the form of an anticlockwise torque about the applicable pallet arm pivot.

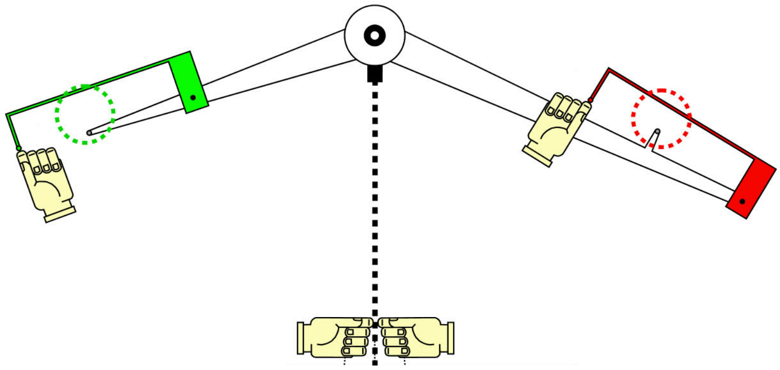


Figure 7.06 - Composers raised (exaggerated), pendulum held in the vertical position.

Figure 7.07 (next) - The entry (left hand) and exit (right hand) pallet arms have been added to the previous configuration, thus completing the escapement frame assembly.

It is worth repeating that the pallet arms must be weighted such that they are tail heavy, and, therefore, continuously generate a clockwise torque about their own pallet arm pivot. That torque rotates the pallet arms until they contact their paired composer nose. In both cases, the clockwise pallet arm torque is arranged to be less than the anticlockwise composer torque. Thereby, each pallet arm and composer pairing produce a combined anticlockwise torque. The 'resting' position of each composer and pallet arm pairing is, therefore, as illustrated in figure 7.07.

As an aside, it should be obvious that the combined mass of the escapement frame, pallet arms, composers and pallet pivot pins will act about the escapement frame arbor as a clockwise torque, purely because, quite obviously, the centre of gravity is to the right of the arbor axis. If considered necessary, the escapement frame to the left of the arbor could be weighted, in order to achieve balance about the escapement frame arbor. To avoid complication, perfect balance will be assumed from this point onwards.

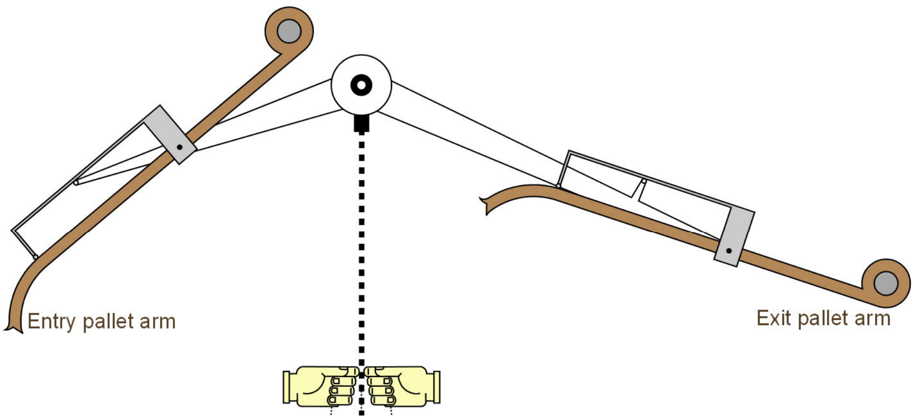


Figure 7.07 - Pallet arms resting upon composers, composers resting upon escapement frame.

It will also be obvious, although illustrated shortly, that deflection of the pendulum will not only rotate the escapement frame about its arbor, but will also rotate the entire assembly of pallet arm pivot pins, pallet arms, composer stops and composers. In such a situation, there would be no relative motion between the frame, pivots, arms, stops or composers. That important principle is emphasised in the next two figures.

Figure 7.08 (next) - The pendulum rod has been pushed and held to the left of vertical, obliging the rigidly attached escapement frame to rotate clockwise about the escapement frame arbor through the same angle. The composers and pallets arms will also rotate about the escapement frame arbor, again by the same angle, but there will be no rotation relative to the pallet arm pivots. Thus, the composers will continue to rest upon the escapement frame and the pallet arms will continue to rest upon their paired composers.

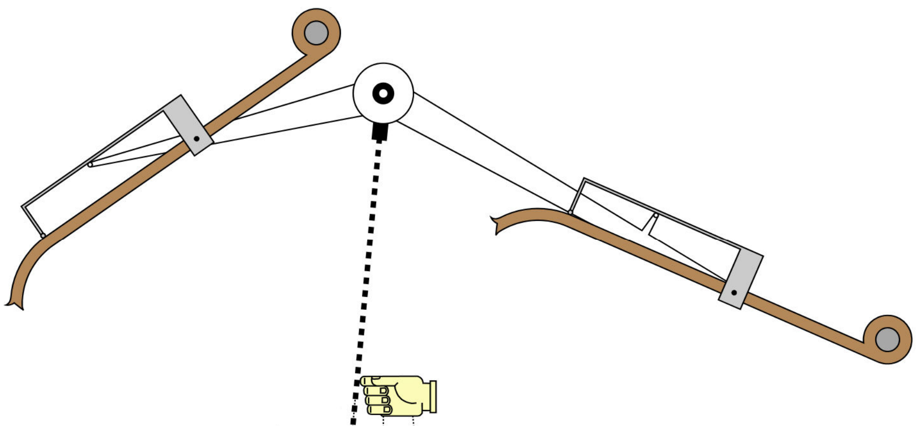


Figure 7.08 - Pendulum rod pushed to the left.

Figure 7.09 (next) - The pendulum rod has been pushed and held to the right of vertical. All components will rotate through the same angle about the escapement frame arbor, with absolutely no relative motion.

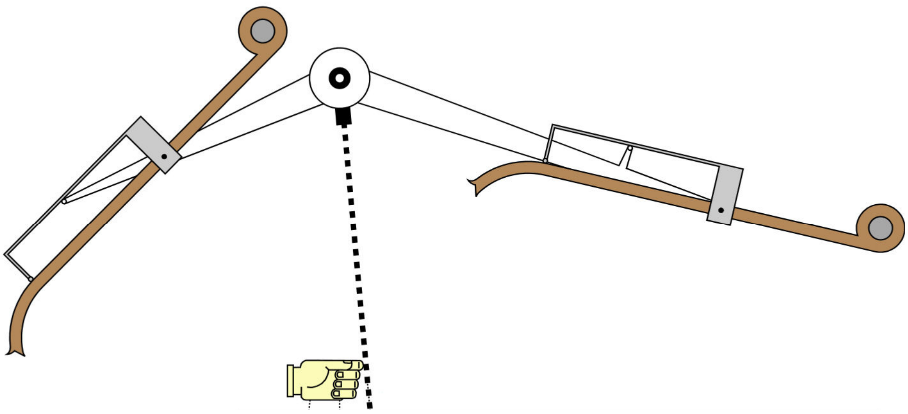


Figure 7.09 - Pendulum rod pushed to the right.

Figure 7.10 (next) - Shows the consequences of applying lift to the **'nib'** ends of the pallet arms. Sufficient lifting force will overcome the overall anticlockwise torque generated by the entry pallet arm and composer pairings. The broken circles indicate that each composer has been lifted out of contact with the escapement frame stops, whilst the pallet arm and composer remain in contact with each other, by virtue of the nose-heaviness of the composers.

As an aside, should the amount of either lift be taken beyond design limits, contact between the pallet arm and the escapement frame might occur. The effects of such treatment could include the disturbance of correct escapement adjustment or, in extreme cases, component damage.

Observe that during rotation there is no relative motion between either pallet arm and its paired composer, by virtue of paired components being mounted upon a common, pallet pivot. Consequently, there is no sliding between paired components and, therefore, no sliding friction, wear or requirement for lubrication.

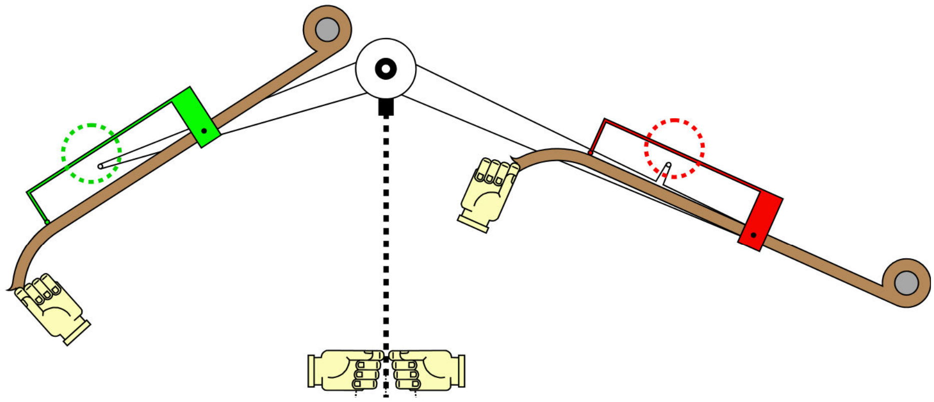


Figure 7.10 - Effects of raising the pallet arms at the nib ends.

Figure 7.11 (next) - Demonstrates the consequences of depressing the nib ends of the pallet arms, or raising the tail ends (for either action has the same physical outcome).

Sufficient depression of a nib (or raising of a tail) will overcome the clockwise torque generated by the tail heaviness of the pallet arm. The broken circles indicate that the pallet arms have rotated out of contact with their paired composer, whilst each composer rests upon its escapement frame stop, by virtue of inherent nose-heaviness.

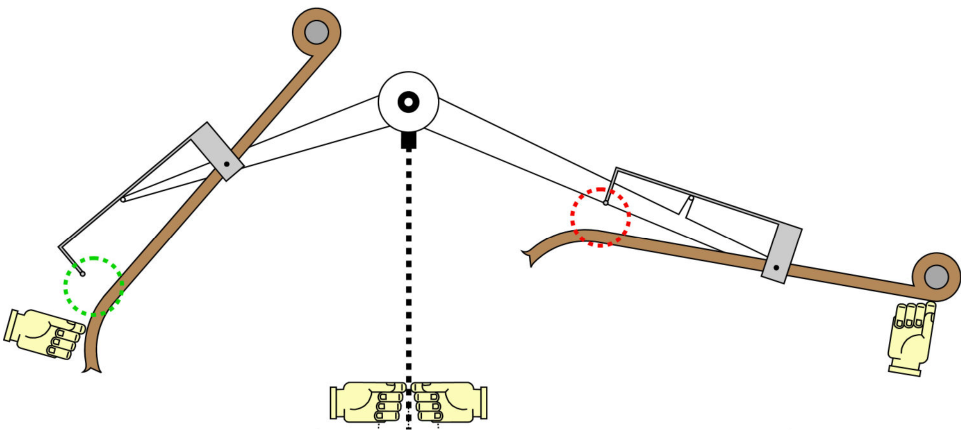


Figure 7.11 - Effects of depressing the pallet arms at the nib ends (or raising the tail ends).

SETTING THE ESCAPEMENT IN MOTION

The sequence of operations involved in setting the twin pivot grasshopper escapement in motion will now be explained, starting with an unwound clock, completely at rest, with the pendulum vertical. The required sequence is identical to part of the normal operating cycle, with the obvious exception that various manual inputs must be made at appropriate stages. The grasshopper demands particular care during what is a *potentially* hazardous operation, in order to avoid complete detachment of the pallets from the escape wheel (referred to as **'trip'**), permitting hazardous, high speed free running of the escape wheel (**'runaway'**), described shortly.

Figure 7.12 (next) represents the complete escapement with the escapement frame connected rigidly to the pendulum. Below the escapement frame assembly lies the escape wheel, which normally rotates clockwise about the escape wheel arbor (an 'anchored pivot'), out of view, at the centre of the escape wheel. The crossed arrow within the rim of the escape wheel indicates that the escape wheel is stationary. A note confirms that there is no torque (turning effort) applied to the escape wheel arbor. The pendulum is being held stationary, in the vertical position, by an imaginary assistant.

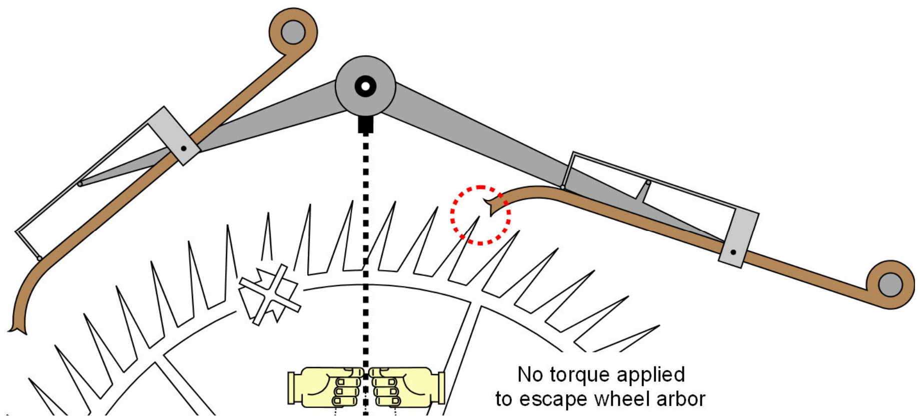


Figure 7.12 - Escapement and escape wheel at rest, pendulum vertical.

A few observations and comments will be of value at this point, before describing the sequence of setting the escapement in motion in any detail:

The entry pallet nib is well clear of the escape wheel and may be disregarded for now. However, of considerable significance is the position and form of the exit pallet nib, on right hand side. The broken circle highlights both the nib and the closest escape wheel tooth, anticlockwise removed. It should be clear that if clockwise torque was applied to the escape wheel, the tooth would miss the nib, albeit by a small margin. Further escape wheel rotation would, therefore, *not* be prevented. A completely free escape wheel, driven by the weight and train, would rotate at an ever increasing and eventually alarming rate and the driving weight would descend ever more rapidly. This is a dangerous 'runaway' situation, both for the clock and operatives, since it will usually end with the weight crashing to the bottom of the case and damage to the case, movement and/or escapement. Furthermore, should the escape wheel be in a runaway situation and either pallet nib move into the path of the speeding teeth, it is almost inevitable that damage to the nib would occur and it is possible that the escape wheel teeth could also sustain damage. There is, therefore, a clear incentive to ensure that the *exit* pallet nib is longer than the illustrated form, provided that other factors, such as correct and reliable escapement functioning, clearance between relevant components etc. are not affected. For the remainder of this publication, the pallet nibs will remain as shown, serving to illustrate the care demanded by certain operations.

For the illustrated escape wheel 'count' (total number of teeth), the resting *entry* pallet nib is of little value in preventing, or even minimising, the risk of escapement trip.

There can be no doubt that the grasshopper is superior to many other types of escapement. However, the previous warnings serve to highlight one major weakness: incorrect construction and careless or ignorant handling can be responsible for the cause of severe damage due to trip and runaway. No doubt as a consequence of that vulnerability, few original Harrison pallet arms have survived to this day. Indeed, the Brocklesby clock itself has been the victim of incorrect operation, necessitating some repair of the pallet arms on one occasion. It is conceivable that many horologists have, very unfairly, interpreted a susceptibility to careless handling as sufficient reason to avoid the grasshopper in favour of escapements of significantly poorer performance.

Figure 7.13 (next) - The pendulum is vertical and no torque is applied to the escape wheel. The exit pallet arm has been manually lifted at the tail end and the escape wheel rotated, until the exit nib locking corner engages with an escape wheel tooth tip, highlighted by the broken circle at the exit nib. The exit pallet arm will thereby lose contact with the exit composer, identified by the broken circle at the nose of the exit composer. The engagement operation is slightly more awkward to execute than for the single pivot grasshopper escapement, although difficulties could certainly be minimised by extending the pallet nib, whilst maintaining the position of the locking corner. Observe that he escape wheel tooth in contact with the exit pallet is illustrated with a dark infill, which will ease the process of understanding the remaining sequence.

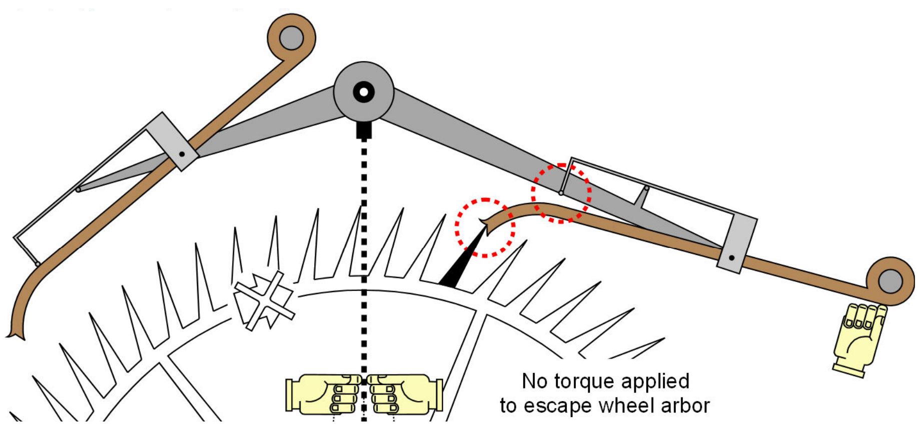


Figure 7.13 - Exit pallet tail lifted, as necessary. Exit pallet nib locking corner manually engaged with an escape wheel tooth tip.

Figure 7.14 (next) - Torque has been applied to the escape wheel, by winding the clock, thereby raising the driving weight and supplying energy to the movement. A note within the escape wheel confirms the application of torque. Further escape wheel rotation is now prevented, by virtue of the dark engaged escape wheel tooth tip being locked, albeit in a clockwise direction only, by the locking corner of the exit pallet nib.

In passing, if the pendulum was to be released, the torque from the escape wheel would feed through the exit pallet arm and escapement frame, generating an anticlockwise torque about the escapement frame arbor. An unrestrained, stationary pendulum would, therefore, be deflected very slightly to the right of vertical until a balancing clockwise torque was generated by the pendulum mass. The torques and forces involved are normally extremely small and are, therefore, insufficient to displace the stationary pendulum by more than a small amount. Note that the arrow within the rim of the escape wheel is crossed, indicating that the escape wheel is unable to rotate, despite the continuous application of torque from the train.

Provided that sufficient torque to the escape wheel has been applied ***and is maintained***, we may now, as illustrated, remove the manually applied force previously positioning the exit pallet locking corner onto the dark escape wheel tooth. Static friction between the dark escape wheel tooth tip and the exit pallet locking corner will, if of sufficient magnitude, hold the exit pallet nib in position, overcoming the tail-heaviness of the pallet arm. The exit pallet locking corner has been **'captured'** by the engaged escape wheel tooth tip.

Despite the aforementioned disastrous consequences of escapement trip and escape wheel runaway, it is essential that the operator remove all manual inputs to the exit pallet arm after nib capture. Unlike the single pivot grasshopper, continued input is far too difficult to correctly sustain during subsequent operations.

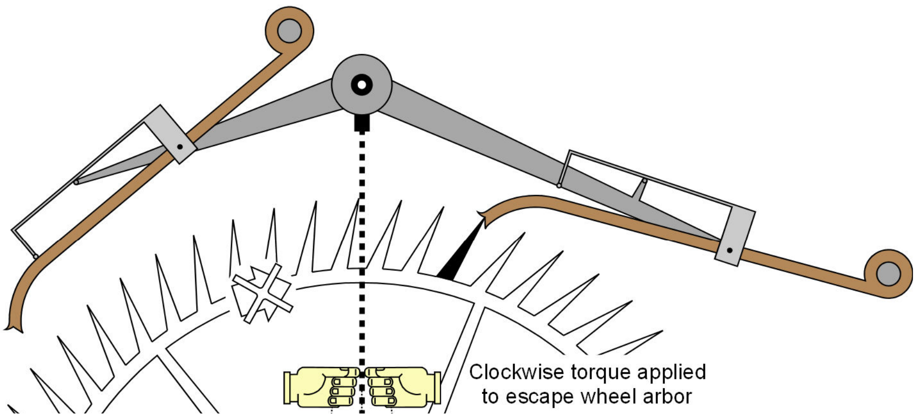


Figure 7.14 - Torque applied by the driving weight, via the movement train, to the escape wheel. Exit pallet nib locking corner captured.

Figure 7.15 (next) - Our assistant is applying a force to the pendulum, to the right, as indicated by the symbolic pushing hand. As a result of the applied force, the escapement frame will rotate anticlockwise about the escapement frame arbor, indicated by the curved arrow.

The exit pallet locking corner will remain captured by the engaged dark escape wheel tooth (continued capture being indicated by the small red circle at the exit pallet locking corner) and the exit pallet arm will begin to rotate anticlockwise about its pallet arm pivot pin. In doing so, the exit pallet arm will break contact with the exit composer, which is restrained by the exit 'stop'. The escape wheel will begin to rotate clockwise.

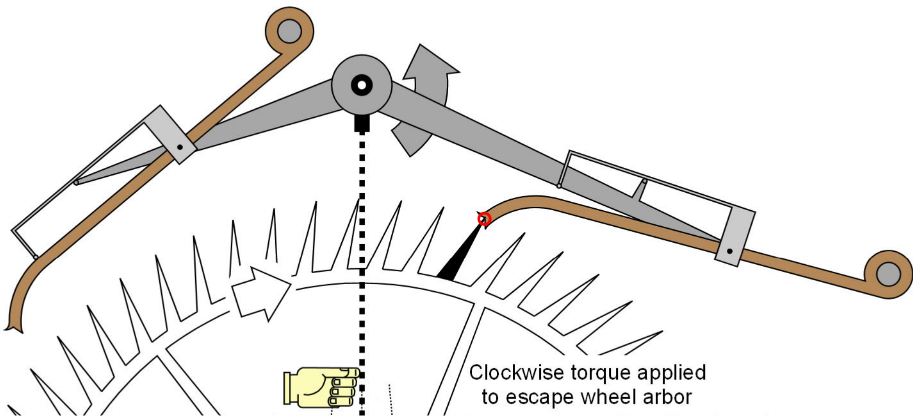


Figure 7.15 - Commencement of manual pendulum displacement to the right. Exit pallet impulsing.

Figure 7.16 (next) and Figure 7.17 (next but one) - These separate figures represent two events occurring virtually simultaneously. However, the process of complete understanding requires that they be described separately.

Figure 7.16 (next) - Our assistant has continued to push the pendulum to the right, thereby rotating the escapement frame further anticlockwise. The entry pallet arm and both composers rotate in unison with the escapement frame, but the exit pallet arm, still captured by the escape wheel, is obliged to rotate further anticlockwise about its pivot pin.

Eventually, as shown, the entry pallet will contact the escape wheel, the geometry being so carefully designed as to ensure that contact between the pallet nib and an escape wheel tooth tip occurs at precisely the entry pallet locking corner. There may be an audible "click" upon contact, subject to the exuberance of our assistant. Due to the continued movement of the entry pallet pivot towards the escape wheel, the entry pallet will instantaneously prevent further clockwise rotation of the escape wheel, as indicated by the crossed arrow within the escape wheel rim.

At the instant the escape wheel is halted, the entry pallet locking corner is 'captured' by static friction between it and the engaged escape wheel tooth tip.

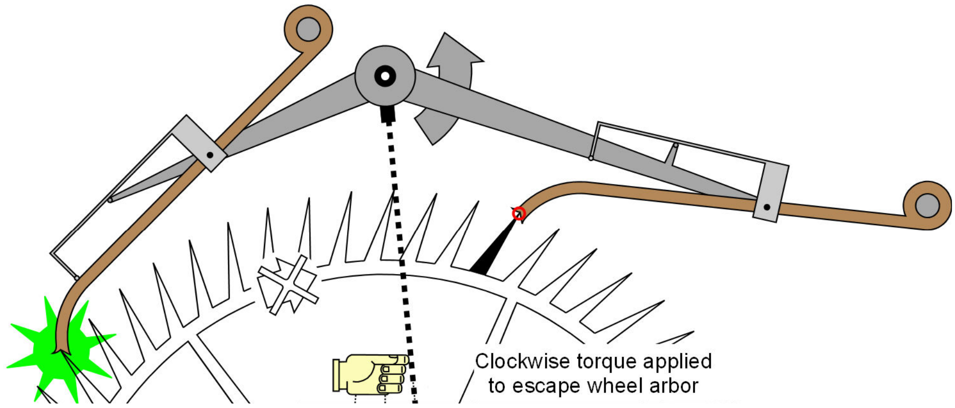


**Figure 7.16 - Entry pallet locking corner contacts the escape wheel.
Entry pallet nib captured. Escape wheel halted.**

Figure 7.17 (next) - The pendulum has been pushed further to the right by an *imperceptible* amount. The escapement frame has rotated further anticlockwise, moving the entry pallet arm pivot pin closer to the escape wheel and obliging the entry pallet arm to rotate clockwise about that pivot. The only means by which the escapement mechanism can absorb such movements is to force the escape wheel into recoil, as indicated by the anticlockwise arrow within the escape wheel rim. As another consequence of the clockwise rotation of the entry pallet arm, the entry composer becomes detached from the escapement frame, albeit by an imperceptible amount (note the change in the illustrated colour of the composer, from grey to green, indicating that it is no longer 'resting' on its stop.)

Recoil of the escape wheel **'releases'** the exit pallet nib, by virtue of removing the applied force and, therefore, the generated friction. The tail heavy exit pallet arm is then free to rotate clockwise about its pivot, away from the escape wheel.

The capture of the entry pallet nib has been entirely responsible for the release of the exit pallet, the time interval between entry pallet capture and exit pallet release being extremely short. In fact, for all practical purposes, the two events occur simultaneously.

Shortly after exit pallet release, the exit pallet arm will contact the exit composer and rest upon it. Subject to the degree of tail weighting of the exit pallet arm, the motion and contact could be relatively slow and gentle, generating almost no sound, or it could be rapid and hard, producing a clearly audible "click". There might even be some bounce of the pallet upon contact, creating a series of "clicks" of diminishing volume. In extreme cases, contact could also induce bounce of the composer on the escapement frame, creating further sounds. Note carefully that, unlike the anchor and dead beat escapements, such sounds do not correspond to first contact between an escape wheel tooth tip and a pallet and would, therefore, be of no assistance when setting the clock 'in beat'.

In the above situation, the newly captured entry pallet is now receiving impulse from the escape wheel. That impulse acts along a line joining the entry pallet locking corner to its pivot pin, which creates a clockwise torque about the escapement frame arbor, attempting to swing the pendulum from right to left.

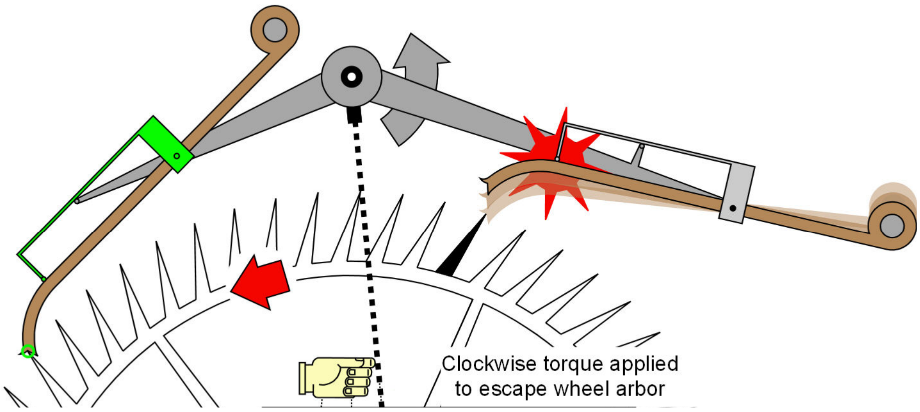


**Figure 7.17 - Virtually coincident with Figure 7.16.
Escape wheel recoil begins. Entry composer begins to lift. Exit pallet nib released.**

Figure 7.18 (next) - The pendulum displacement to the right continues, slightly beyond the right hand displacement marker (i.e. into **'overswing'**), by virtue of pendulum momentum.

In normal operation, during overswing, the momentum of the pendulum is gradually overcome by the opposing impulse from the newly captured entry pallet locking corner. The entry pallet arm is forced to rotate further clockwise about its pivot pin and the escape wheel will continued to recoil. A further outcome is that the entry composer is obliged to rotate about the common pallet pivot in unison with the entry pallet, thereby lifting the composer out of contact with the escapement frame entry stop, emphasised in the magnified view. By that means, the motion of the escapement frame during overswing is absorbed by the escapement mechanism.

The start-up sequence has now been completed. The pendulum would usually be released and any further manual inputs avoided, permitting uassisted operation of the escapement, which should continue provided that sufficient torque is applied to the escape wheel. The complete cycle of events during the normal cycle of operation will be described in the next section. The sequence just described represents one method of starting a timepiece fitted with a twin pivot grasshopper escapement. Slight variations in approach may be possible, although any alternative methods must ensure that escapement trip and escape wheel runaway are prevented.

It should now be clear that the starting of a stationary clock with a twin pivot grasshopper escapement is slightly more involved than the common method of winding the key without preparation, followed by indiscriminate swinging of the pendulum. Such an approach will lead to certain disaster.

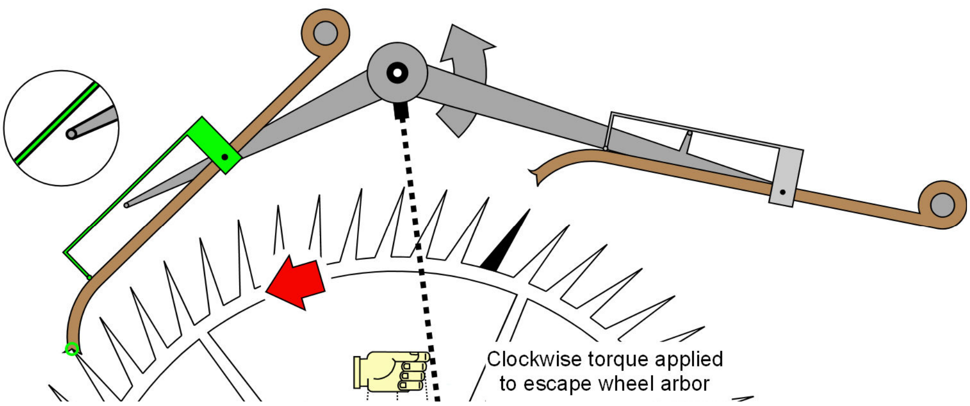


**Figure 7.18 - Pendulum overswing. Escape wheel recoil continues. Entry pallet still captured.
Entry composer visibly lifted clear of escapement frame.**

CYCLE OF OPERATION

Figure 7.19 (next) - With thanks, our helpful assistant may now enjoy a well-earned rest. From this point onwards, provided that the driving weight is of sufficient size, a correctly constructed and adjusted escapement will allocate sufficient energy to the pendulum to maintain a continuous cycle of escapement operation, rotation of the movement train and advancement of the indicating hands. That complete cycle of operation will now be described and explained in detail.

The pendulum is at the limit of overswing to the right, at the start of its journey from right to left and the rigidly connected escapement frame is rotating clockwise about the escapement frame arbor. The entry pallet locking corner is receiving impulse from the escape wheel and the entry pallet arm is rotating anticlockwise about its pivot pin. The escape wheel impulse to the entry pallet locking corner acts along a line joining the corner to the entry pallet pivot pin, which creates a clockwise torque about the escapement frame arbor, thereby assisting gravity in swinging the pendulum from right to left. The entry composer is detached from the escapement frame, as emphasised in the magnified view, but begins to rotate, with the entry pallet arm, anticlockwise about its pivot, moving it towards the frame. Note carefully the position of the dark escape wheel tooth closest to the exit pallet nib. That tooth will record the progress of the escape wheel during a complete cycle of events, starting at the illustrated position.

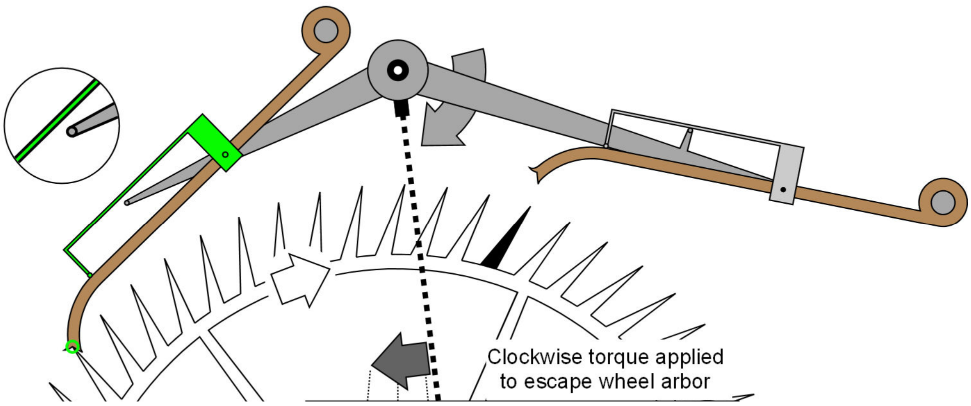


Figure 7.19 - Limit of overswing. Start of entry pallet impulse.

Figure 7.20 (next) - The pendulum has now reached the end of overswing and is, therefore, passing through the right hand amplitude marker. The entry pallet is still receiving impulse from the escape wheel, thus assisting gravity in swinging the pendulum from right to left. The anticlockwise rotations of the entry pallet arm and entry composer about their pivot have only just reached the stage at which the composer has contacted the escapement frame (broken green circle). Sound is rarely generated by such composer contact, which is a "placement", rather than an impact.

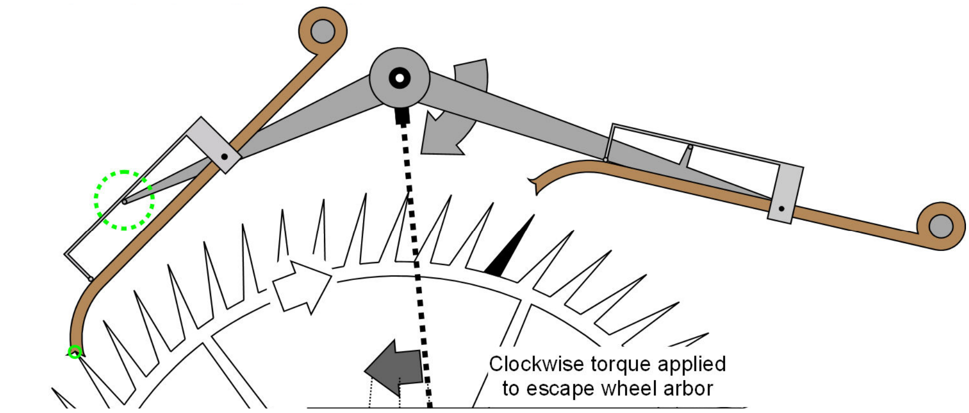


Figure 7.20 - Entry pallet impulsing. Entry composer halted by escapement frame.

Figure 7.21 (next) - As soon as the pendulum swings to the left of the position shown in the previous figure, the continued anticlockwise rotation of the entry pallet about the pallets pivot will lead to a loss of contact with the entry composer, which is left behind, resting upon the escapement frame stop. As illustrated, the pendulum is passing through mid-swing and the entry pallet arm has rotated away from the composer to an obvious degree, emphasised by the broken circle. The resting exit pallet arm nib and composer pairing are moving in unison with the escapement frame, which is rotating clockwise about the escapement frame arbor.

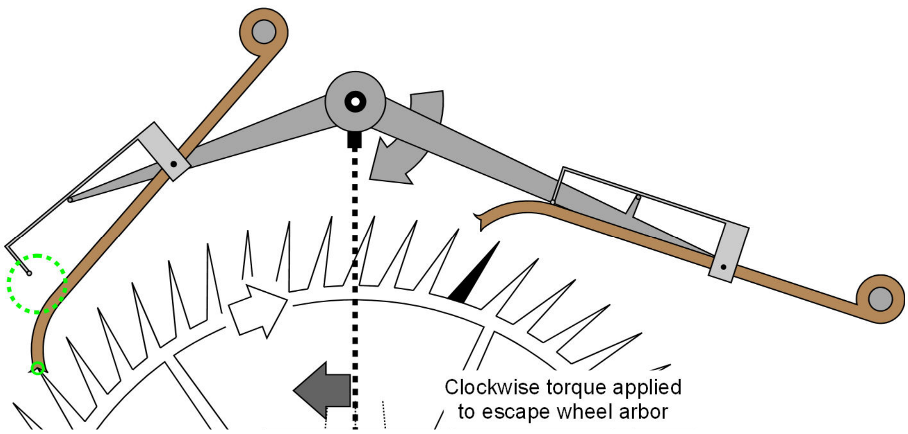


Figure 7.21 - Entry pallet impulsing. Entry pallet arm detached from composer.

Figure 7.22 (next) and **Figure 7.23 (next but one) -** These separate figures represent two events occurring virtually simultaneously. However, the process of complete understanding requires that they be described separately.

Figure 7.22 (next) - The exit pallet arm has rotated with the escapement frame and has just come into contact with the escape wheel, the geometry having been arranged such that contact between the pallet nib and escape wheel tooth tip occurs at precisely the exit pallet locking corner. A "click" sound may be generated. The escape wheel is momentarily halted and the exit pallet locking corner is captured

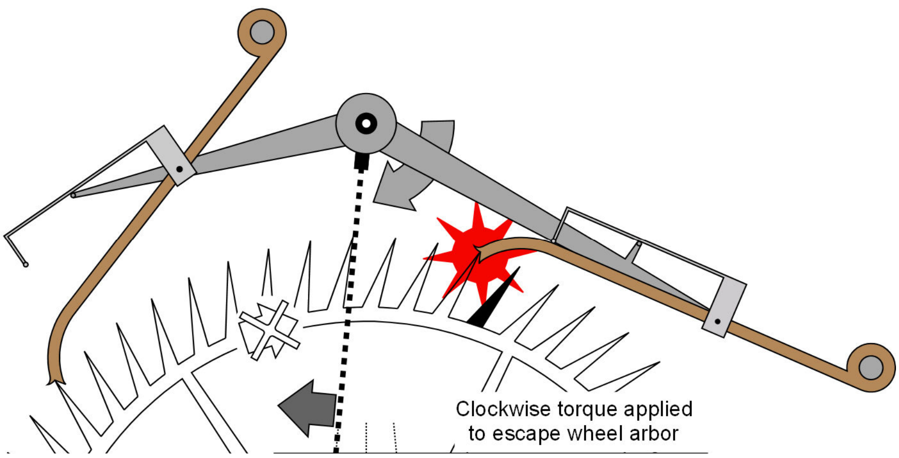


Figure 7.22 - Exit pallet nib locking corner capture. Escape wheel halted. End of entry pallet impulse.

Figure 7.23 (next) - Escape wheel impulse to the exit pallet applies anticlockwise torque to the escapement frame arbor. Diminishing pendulum momentum opposes that torque during overswing to the left.

The exit pallet arm begins to rotate clockwise about its pallet pivot pin, which also begins to rotate the exit composer clockwise, in unison. The exit composer begins to separate from its escapement frame stop (illustrated composer colour changes from grey to red).

The escape wheel is recoiled by the exit pallet, which releases the entry pallet. The released entry pallet rotates clockwise about its pivot until arrested by the entry composer. There may be an audible "click" as the entry pallet arm and composer meet. There may be further clicks if there is pallet arm bounce and even more if there is composer bounce. The extent and severity of bouncing depends upon component masses and the extent to which the exit pallet is tail heavy and the composer is nose heavy.

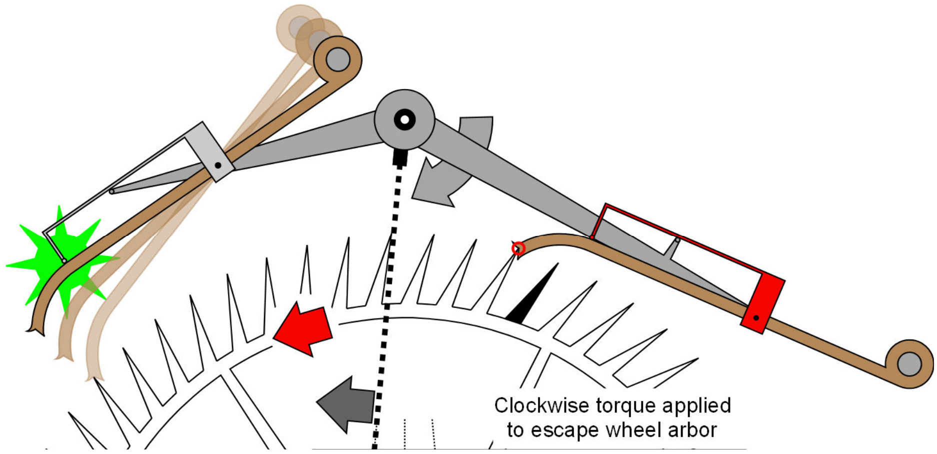


Figure 7.23 - Virtually coincident with Figure 7.22. Escape wheel recoil begins. Entry pallet nib released. Exit composer begins to lift.

Figure 7.24 (next) - Pendulum overswing and escape wheel recoil. Diminishing pendulum momentum and persistent exit pallet impulse act in opposition, until the pendulum and escape wheel both stop, momentarily, at the limit of overswing to the left. The exit composer has been rotated further clockwise about its pivot by the exit pallet arm, separating it from the escapement frame, as emphasised in the magnified view.

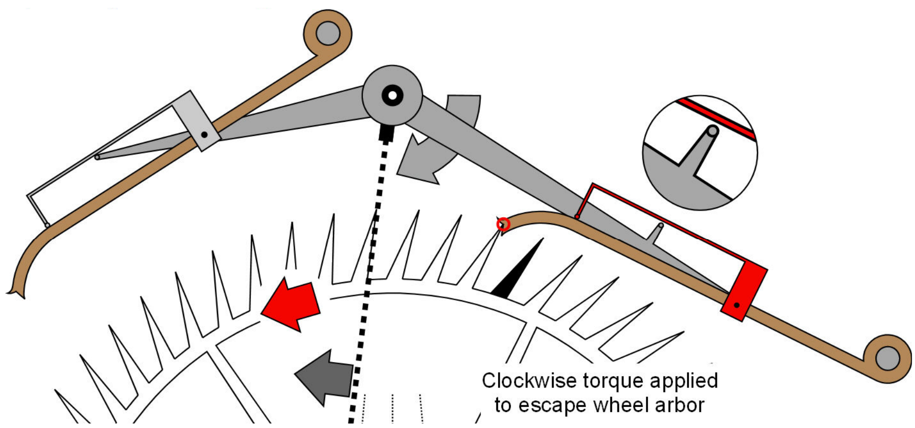


Figure 7.24 - Pendulum overswing. Escape wheel recoil continues. Exit pallet remains captured. Exit composer visibly lifted clear of escapement frame.

Figure 7.25 (next) - The pendulum begins to swing to the right and the escapement frame rotates anticlockwise. The exit pallet arm is receiving impulse from the escape wheel and begins to rotate anticlockwise about its pivot. The exit composer is still clear of the escapement frame, but begins to rotate anticlockwise with the exit pallet arm, about their common pivot, moving it closer towards contact with the escapement frame stop.

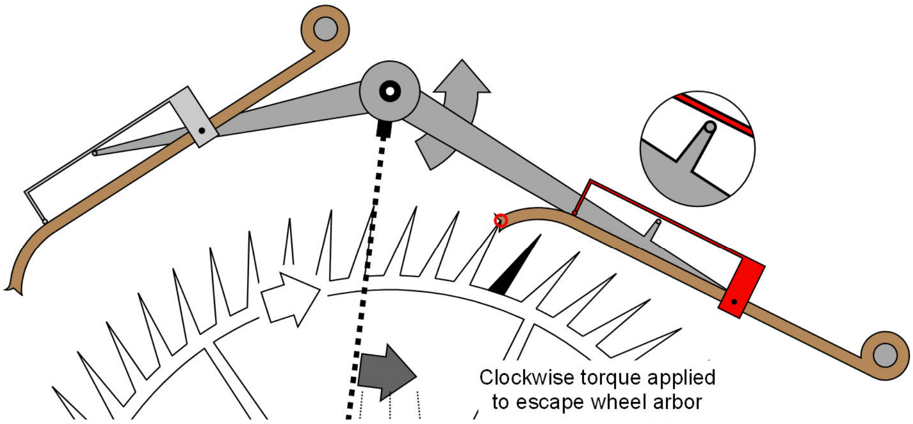


Figure 7.25 - Limit of overswing. Start of exit pallet impulse.

Figure 7.26 (next) - The pendulum has reached the end of overswing and is passing through the left hand amplitude marker. The exit pallet is still receiving impulse from the escape wheel, thereby assisting gravity in swinging the pendulum from left to right. The anticlockwise rotations of the exit pallet arm and entry composer about their pivot have only just reached the stage at which the composer has contacted the escapement frame stop, at the location highlighted by the broken circle. Sound is rarely generated, it being a composer "placement", rather than an impact.

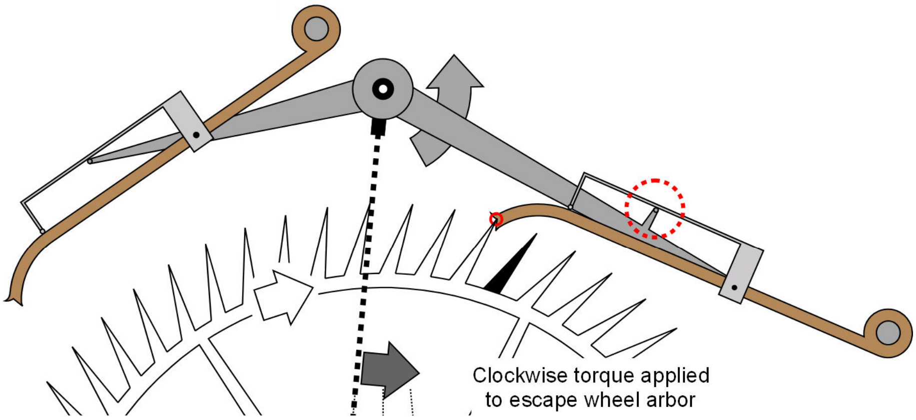


Figure 7.26 - Exit pallet impulsing. Exit composer halted by escapement frame.

Figure 7.27 (next) - The exit pallet arm continues to rotate anticlockwise about its pivot pin, obliging it to break contact with the exit composer, which is left behind, resting on the escapement frame stop. As illustrated, the pendulum is passing through mid-swing and the exit pallet arm is, by then, well clear of the composer. Note the position of the entry pallet nib, compared to the previous figure: it is being rotated, in unison with the entry composer and escapement frame, about the escapement frame arbor, towards the escape wheel.

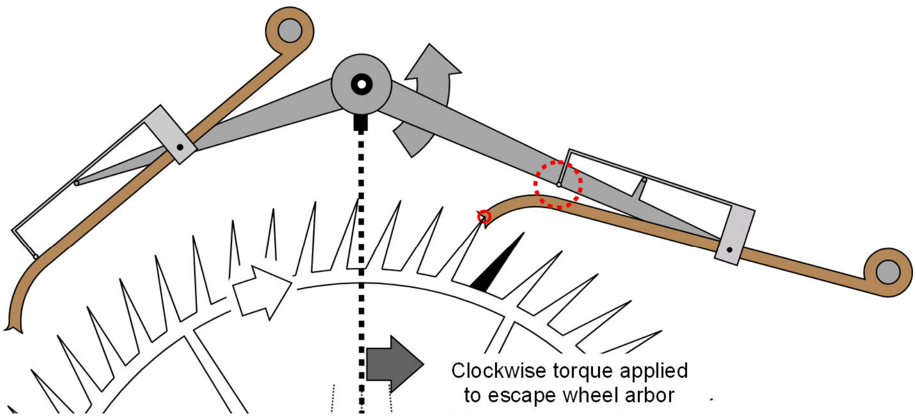


Figure 7.27 - Exit pallet impulsing. Exit pallet arm detached from composer.

Figures 7.28 (next) and 7.29 (next but one) occur virtually simultaneously, although they are illustrated and described separately, in order to ensure complete understanding.

Figure 7.28 (next) - As the pendulum passes the right hand amplitude marker, the entry pallet locking corner contacts an escape wheel tooth tip. There may be an audible "click" on contact. The escape wheel is briefly halted and the entry pallet nib locking corner is captured.

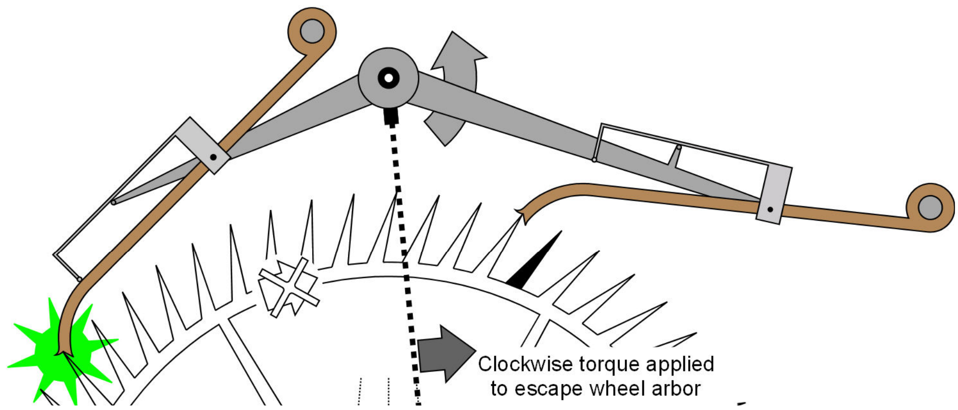


Figure 7.28 - Entry pallet nib locking corner capture. Escape wheel halted. End of exit pallet impulse.

Figure 7.29 (next) - Escape wheel impulse to the entry pallet applies clockwise torque to the escapement frame arbor. Diminishing pendulum momentum opposes that torque, during overswing to the right.

The entry pallet arm begins to rotate clockwise about its pallet pivot pin, which also begins to rotate the entry composer clockwise, in unison. The entry composer begins to separate from its escapement frame stop (illustrated composer colour changes from grey to green).

The escape wheel is recoiled by the entry pallet, which releases the exit pallet. The exit pallet rotates clockwise about the pallets pivot until arrested by the exit composer. There may be an audible "click" as the exit pallet arm and composer meet. There may be more clicks if there is bounce.

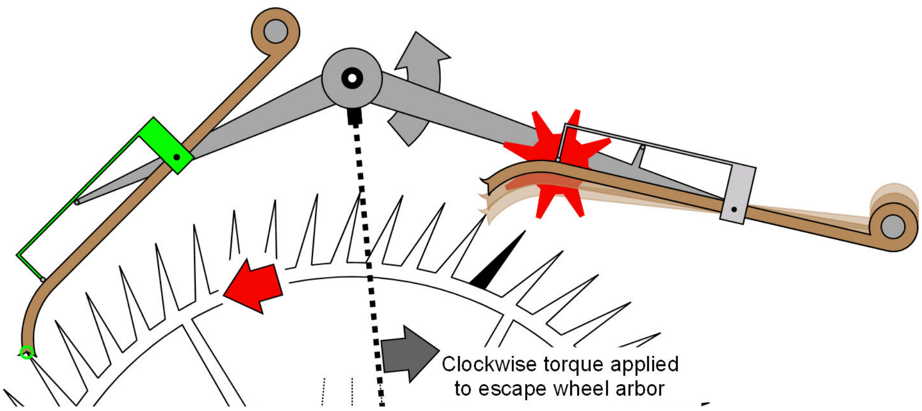


Figure 7.29 - Virtually coincident with Figure 7.28.
Exit pallet arm released, escape wheel recoil. Entry composer begins to lift.

Figure 7.30 (next) - Pendulum overswing and escape wheel recoil. Pendulum momentum opposes entry pallet impulse until the pendulum and escape wheel eventually stop, momentarily, at the limit of overswing. The entry composer has rotated clockwise about its pallet pivot, visibly lifting it out of contact with the escapement frame.

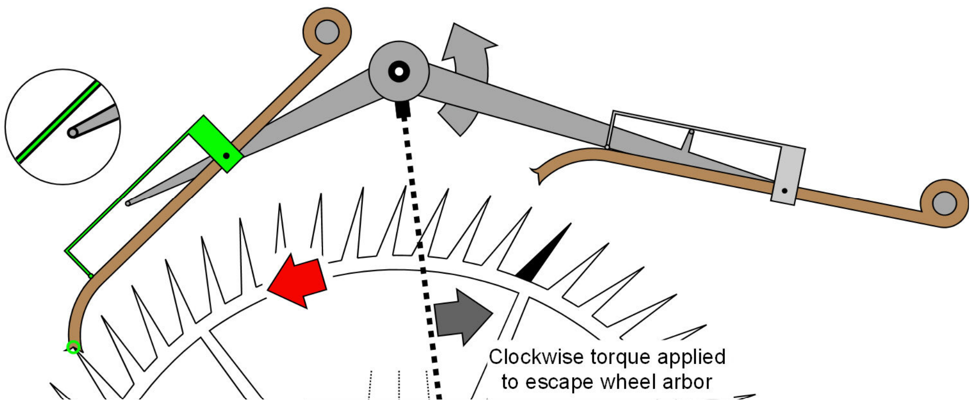


Figure 7.30 - Pendulum overswing. Escape wheel recoil continues. Entry pallet still captured.
Entry composer visibly lifted clear of escapement frame.

CYCLE OF OPERATION
TWO MINUTE ESCAPE WHEEL

The sequence of Figures 7.19 to 7.30 spans one complete cycle of operation, during which the dark escape wheel tooth has advanced clockwise by one full tooth space. In normal operation, one complete cycle of operation would require two swings of the pendulum, i.e. from one extremity to the other and back again, which would occupy two seconds, when executed by a seconds beating pendulum. There are 60 teeth in the escape wheel, which will, therefore, require 2 x 60 = 120 seconds (i.e. two minutes) to complete one full rotation. This may be conveniently referred to as a ''two minute escape wheel'. Please note that the use of a two minute escape wheel for the illustrations should not be taken to imply that it is the optimum. Optimisation will be discussed in a later section.

CYCLE OF OPERATION
NO FRICTION, LUBRICATION OR WEAR

If we now review all of the described stages involved in normal operation, we arrive at the startling conclusion that sliding friction is at all times completely absent. There is negligible sliding contact at the outer surface of each pallet pivot, as the pallet arms and composers rotate through small angles, during the capture and release phases. Practical experience, which is ultimately the real test, confirms that the pivot pins and pallet arms should be capable of operating continuously for many centuries without lubrication or noticeable wear, given suitable choices of materials and correct manufacture. Here, then, is an escapement requiring absolutely no maintenance whatsoever and suffering virtually no wear over centuries of operation. It is sobering to consider that a century of continuous service would involve just over 3155 million beats of the pendulum and that we could expect reliable, continuous operation for at least three of those centuries without lubrication, maintenance or any degradation in function.

PERFORMANCE STIPULATIONS

In 1775, the year before his death, Harrison recorded much of his life's work in a remarkable and immensely significant manuscript entitled :

'A DESCRIPTION CONCERNING SUCH MECHANISM AS WILL AFFORD A NICE, OR TRUE MENSURATION OF TIME; TOGETHER WITH SOME ACCOUNT OF THE ATTEMPTS FOR THE DISCOVERY OF THE LONGITUDE BY THE MOON; AS ALSO AN ACCOUNT OF THE DISCOVERY OF THE SCALE OF MUSIC'.

That document will henceforth be referred as **'CSM'** (derived from ‘CONCERNING SUCH MECHANISM’).

CSM, included detailed stipulations for the design of all future timepieces adhering to Harrison’s principles. Included amongst those stipulations were precise requirements for the performance of his grasshopper escapement.

As with much of his work, Harrison gives scant proof, if any, of his conclusions, many of which were almost certainly derived from practical experience and experimentation. It should not be inferred from this that he was lacking mathematical ability, for that is certainly not the case, but it does serve to explain his tendency to state conclusions without supporting, written evidence. In fairness, Harrison's willingness to devote time and effort to the production of manuscripts in any form is to be commended and we should be grateful for what we have. Nevertheless, to those who understandably find Harrison's approach difficult to accept, it may be reassuring to learn that, in all of his work, he demonstrated complete devotion to establishing absolute truth. In some instances, most notably in his efforts to create a viable longitude timepiece, his ruthless honesty was, it could be argued, to his own severe disadvantage, most especially in financial terms. At this stage in this publication, therefore, we shall accept Harrison's CSM statements, until evidence or doubts to the contrary arise. As will be appreciated, a remarkable aspect of the twin pivot grasshopper escapement is that it may be adapted to satisfy virtually any realistic performance requirements, if desired. Our adherence to Harrison's CSM stipulations is, therefore, by no means irreversible.

It should be bourne in mind that many other CSM stipulations, encompassing all aspects of the design, setup and operation of an entire Harrison regulator, must be met in order to create a timepiece capable of the intended performance. In short, any grasshopper escapement, however perfect it may be, would be less effective and quite possibly inappropriate, in isolation from Harrison's other devices.

A relevant summary of Harrison's CSM stipulations is listed next. The numerical ordering is peculiar to this publication, for ease of reference, and should, most emphatically, neither be attributed to Harrison nor assumed to indicate any particular order of importance. Other stipulations, requirements and statements are presented in CSM; however, since they have no *currently* apparent influence upon the design or optimisation of the escapement, they will not be listed or described. We must, however, be receptive to, but cautious of, future interpretations of CSM that may conflict with this view; CSM can be intricate and may well hold many undiscovered secrets, but that intricacy and potential renders it susceptible to erroneous, misleading and/or deliberately false interpretations.

The reader may, quite understandably, be somewhat daunted and, perhaps confused by two of the stipulations listed below, numbered 2 and 3. A subsequent section, entitled TORQUE ARMS, RATIOS AND CIRCLES, will carefully explain their meaning at an arguably more appropriate point.

- **Stipulation 1 - There must be no sliding friction and no requirement for lubrication.**

This achievement has already been demonstrated and explained in the previous section.

- **Stipulation 2 - The mean torque arm must be one hundredth of the equivalent pendulum length.**

The term 'mean torque arm' requires clarification, which will be presented later, at a more appropriate point. In simple terms, Harrison is restricting the influence of the escapement, in order that the natural, free swing of the pendulum is not interfered with to an excessive degree.

The equivalent pendulum length is defined as the length of an idealised pendulum having point suspension, no rod mass and all pendulum mass concentrated at the centre of a circular bob. The period of such a pendulum must match that of the pendulum fitted to the timepiece in question. The equivalent length is the distance from the suspension point to the centre of the bob. A later section will include further illustration.

- **Stipulation 3 - The mean torque at the start of impulse must be two thirds* (2/3 or 0.6666 recurring) of the mean torque at the end of impulse, over one complete cycle of operation.**

The term 'mean torque' will be explained when appropriate. In simple terms, Harrison is defining the way in which escapement impulse should vary during each swing of the pendulum.

**In fact, CSM is annoyingly (and unnecessarily) imprecise as to the exact ratio to be used, as will be discussed later.*

- **Stipulation 4 - The pendulum arc should be large, although fifteen degrees should not be exceeded.**

For the purposes of this publication, eleven degrees will be assumed to be the ideal. That choice is not irreversible, for the design methods presented herein will permit the specification of any chosen pendulum arc, if so desired. The grasshopper escapement has a rare capacity to work at extremely large amplitudes with absolutely no degradation in performance. Suspension cheeks are, however, essential (and an extremely useful ally).

- **Stipulation 5 - A 'long pendulum' must be incorporated.**

All of Harrison's pendulums were seconds beating (or, to be precise, at or close to seconds beating). Seconds beating pendulum lengths will be assumed herein.

- **Stipulation 6 - A four minute escape wheel shall be used.**

An understanding of this stipulation will require a study of the later OBSERVATIONS section. Please note that this stipulation was almost certainly written with the single pivot grasshopper in mind. However, the arguments also apply to the twin pivot configuration, the use of a two minute wheel being merely for clarity of explanation.

GEOMETRICAL REPRESENTATION

The objective is to create independent graphical and mathematical methods for the design of optimised, CSM compliant, twin pivot grasshopper escapement geometries. The essential first task is to identify defining parameters and combine them within a representative geometry. Harrison's CSM stipulations will be adapted and incorporated as standard, although, of vital importance, any devised methods should also be capable of accommodating alternative constraints, with a view to stimulating and incorporating future research and discoveries.

Any attempt to create a representation of the twin pivot grasshopper configuration reveals that there is little prior information to assist in the design of a geometry complying with CSM. The Brocklesby twin pivot grasshopper escapement clearly fails to comply and is of little value, except as a unique record of the basic mechanical principles. However, a feature of Harrison's single pivot geometries, as recorded in his sheet of drawings, MS 3972/3, is the clear presence of right-angles. Right-angles are useful elements of universal geometrical representation and viable mathematical modelling. It will, therefore, be assumed that the line of action of both pallet arms should be at right-angles to the relevant escape wheel radial at the start of impulse.

Further analysis reveals that the twin pivot layout is, in some respects, considerably less co-operative than the single pivot configuration, in more ways than one. Such difficulties may be surprising, given that the single pivot grasshopper is mechanically more convoluted than the twin pivot. However, it is the common pallets pivot of the single pivot grasshopper, largely responsible for mechanical complexity, that provides the indispensable link between the entry and exit geometries. In contrast, the twin pivot grasshopper incorporates entry and exit geometries possessed of no immediately obvious link(s), apart from a sharing of the escape wheel, escape wheel arbor and escapement frame arbor and a requirement that each side must generate the same pendulum arc.

Fortunately, a solution to this apparently incurable dilemma has been devised, which, at first sight, would appear to add considerably to the difficulties, rather than ease them. As is so often the case with an ‘insoluble problem’, the solution becomes ‘blindingly obvious', once explained.

In preparation, it should be noted that, for the *single* pivot grasshopper geometry, Stipulation 3 must be based on *mean* torque ratio, due to the asymmetrical impulse characteristics of that configuration (i.e. the torque ratio of the entry pallet always differs from the torque ratio of the exit pallet for all but an infinite escape wheel tooth count). Although Harrison states that such asymmetry is of no consequence, neither does he state that it is of any benefit. Furthermore, asymmetrical torque ratios are not specifically demanded by any of Harrison's CSM stipulations, whilst symmetrical torque ratios are not specifically excluded. Consider, then, a no doubt surprising proposal that we demand nothing less than absolutely symmetrical torque arms. To be more specific, if we applied that proposal to Harrison's CSM Stipulation 3, the outcome would be an escapement with an ***entry*** torque arm of two units at the start of impulse and three units at the end of impulse and an ***exit*** torque arm of two units at the start of impulse and three units at the end of impulse. The MATHEMATICAL DESIGN PROCESS sections, presented later, expand upon and illustrate this concept in greater detail.

Although such an approach might appear to compound the already considerable problems of designing a compliant geometry, in reality it identifies the elusive, essential missing link between the entry and exit sides of the escapement, thereby permitting the development of a viable geometry and, from that, a productive mathematical modelling technique. Here also, almost by accident, arises an opportunity to create a perfect variant of the perfect (CSM compliant) grasshopper, with absolutely symmetrical impulse. Such an achievement might, perhaps, have seemed quite impossible, if demanded from the outset. Necessity has, indeed, been the 'mother of invention'.

Repeated, detailed mathematical modelling of twin pivot geometries with symmetrical torque arms quickly reveals a further constraint, which, as before, would appear to add nothing but considerable difficulty. If it is specified that the entry pallet arm active length (separation between pallet locking corner and pallet arm pivot axis) should equal that of the exit pallet arm, it soon becomes apparent that the mathematical process, most especially iteration, is transformed from ‘difficult and tedious’ to ‘relatively straightforward’. By reducing two variables (entry and exit pallet arm active lengths) to what is, effectively, only one (a single, common, active length), the process of iteration becomes considerably more straightforward.

There would appear to be no operational or mechanical disadvantages to the above proposals. Obviously, should there be a future requirement to incorporate differing pallet arm active lengths, reversion to the 'difficult and tedious' method will be necessary.

Figure 7.31 (below) merely serves as an introduction the geometry to be used in the forthcoming analysis. The illustration incorporates a semi-transparent overlay of the mechanical arrangement at the start of entry pallet impulse. By intention, the geometry matches the proportions of the mechanical form illustrated in Figures 7.01 to 7.30. The geometrical form can only, surely, be described as beautiful, although a full appreciation awaits a complete understanding of function. It is vital to appreciate that the geometry represents a combination of every one of the situations depicted in Figures 7.19 to 7.30, with the exception of overswing. Although the mechanical overlay of Figure 7.31 depicts only one of those situations, it serves to demonstrate what the wonderful geometry of the mathematical universe represents in the parallel physical universe.

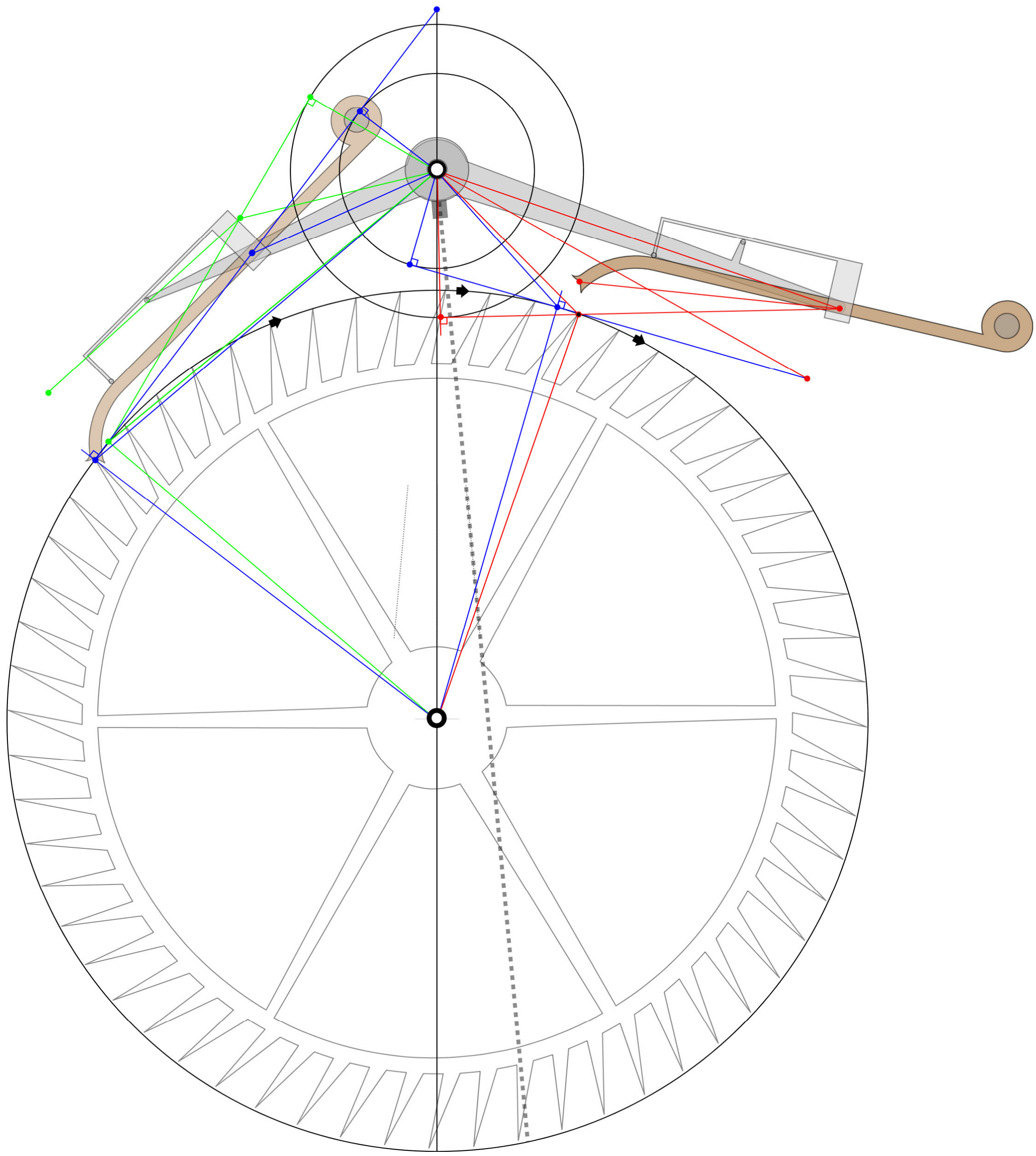


Figure 7.31 - Introductory twin pivot grasshopper escapement geometry with a sample mechanical overlay.

Figure 7.32 (below) shows the geometry to be used in our detailed analyses, with the mechanical overlay removed. The figure has been maximised in size and annotated, in preparation for subsequent descriptions, explanations and mathematical modelling. Wherever possible, green represents new entry side constructions, whilst red represents new exit side constructions. In this particular illustration, however, some constructions are blue, in an effort to avoid confusion between start of impulse and end of impulse geometries. All figures should be regarded as illustrative only.

The generalised geometry confirms that equal entry and exit torque ratios are incorporated, there being only two, common torque arm circles, instead of the four circles required of the asymmetrical impulse single pivot escapement. Equal length pallet arms (locking corner to pivot) are included, because the refined mathematical model output has been used to create the illustrated geometry. Right-angles are incorporated at J and D, as explained in the text. Although the geometry is based on an escape wheel of sixty teeth, that number was chosen entirely because it generated a geometry with excellent clarity. The design methods derived from the geometry will permit any sensible number of teeth to be incorporated. The same flexibility applies to every other designer chosen parameter.

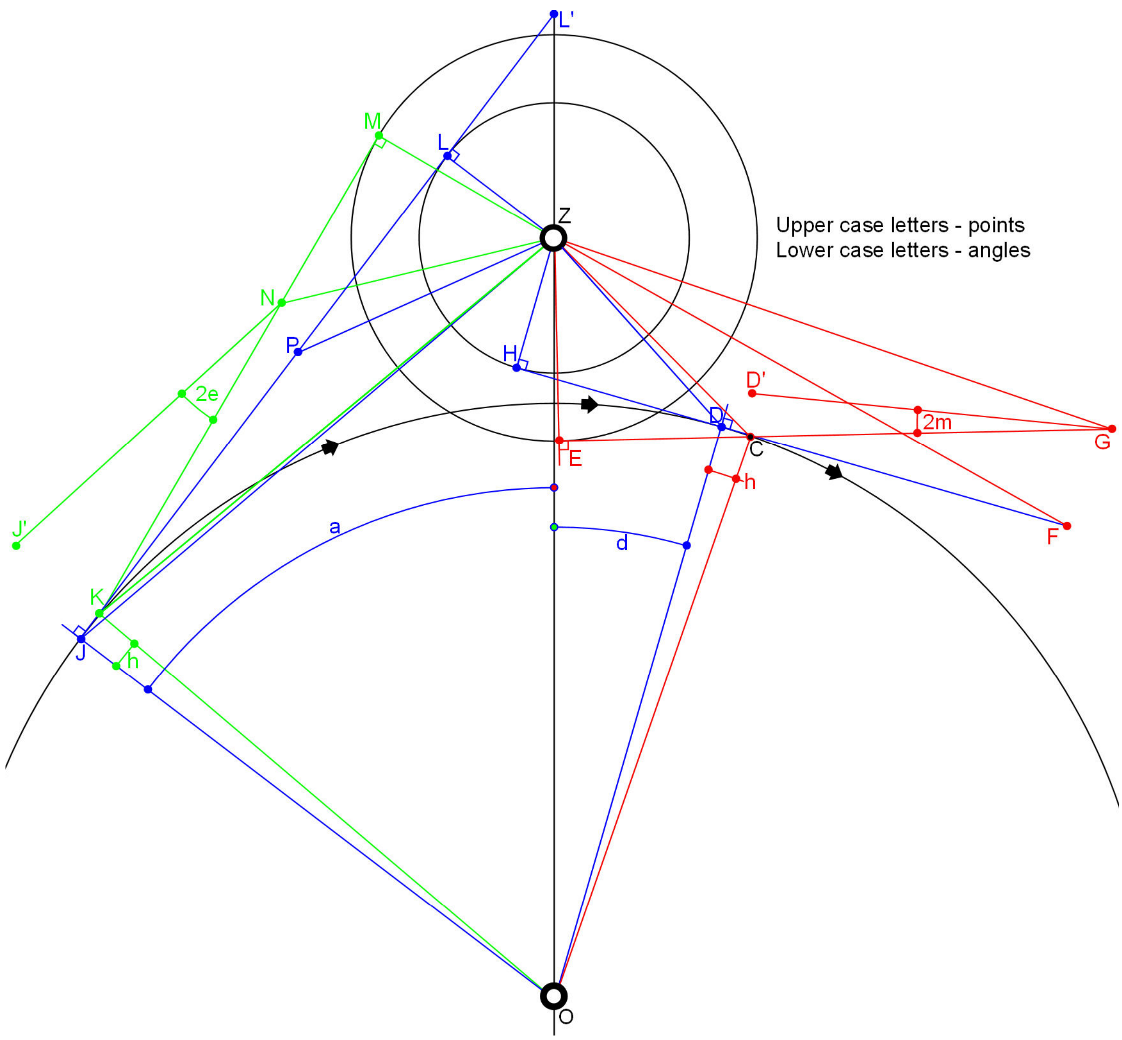


Figure 7.32 - Twin pivot grasshopper escapement geometry.

GUIDE TO FIGURE 7.32 (in alphabetical order)

a +d - The 'span' of the escapement, in degrees.

C - Exit pallet locking corner at the end of impulse (start of release).

CD' - Exit pallet travel ('lift') after release.

Circular arc through , J, K, D and C - locus of escape wheel tooth tips.

D - Exit pallet locking corner at start of impulse (upon capture). DF is perpendicular to DO.

D' - Exit pallet locking corner at the end of release (resting upon exit composer).

e - Entry pallet arm rotates through angle **2e** after release.

EZ - Exit pallet torque arm at the end of impulse.

F - Exit pallet pivot at the start of impulse.

G - Exit pallet pivot at the end of impulse.

h - The angle subtended at O by half a tooth space.

HZ - Exit pallet torque arm at the start of impulse.

J - Entry pallet locking corner at the start of impulse (upon capture). JP is perpendicular to JO.

J' - Entry pallet locking corner at the end of release (resting upon entry composer).

K - Entry pallet locking corner at the end of impulse (start of release).

KJ' - Entry pallet travel ('lift') after release.

LZ - Entry pallet torque arm at the start of impulse.

m - Exit pallet arm rotates through angle **2m** after release.

MZ - Entry pallet torque arm at the end of impulse.

N - Entry pallet pivot at the end of impulse.

O - Escape wheel arbor. Anchored pivot.

P - Entry pallet pivot at the start of impulse.

Z - Escapement frame arbor. Anchored pivot.

TORQUE ARMS, RATIOS AND CIRCLES

ENTRY GEOMETRY

In **Figure 7.32 (shown earlier)**, blue line JP represents the most direct connection between the entry pallet locking corner at the start of impulse (immediately upon capture) and the corresponding position of the entry pallet arm pivot pin. Although the physical pallet arm will obviously differ in form from a simple straight line, its shape is irrelevant to the connection of those two points. Both end points are, effectively, frictionless pivots (ignoring the negligible friction at the pallet arm pivot, as justified in an earlier section). Thus, the component, along JP, of any force applied to one of the end points will be transmitted along JP to the other end point, where it will emerge, unaltered in magnitude or direction by the journey. Any other components of force will not be transmitted.

In order to determine the torque generated about the escapement frame arbor, Z, by the force along JP, we must extend the line of that force until a perpendicular to it passes through Z. In Figure 7.32, line JP is extended (further than strictly necessary) to L' and the perpendicular is shown as the line LZ. The required torque is the product of the force along JP and distance LZ.

LZ is referred to as the **'torque arm'** of the entry pallet arm, JP, at the start of impulse (immediately upon capture).

By the same reasoning, MZ is the torque arm of the same entry pallet arm, now represented as KN, at the end of impulse (at the time of release).

The **'torque arm ratio'** for the entry pallet arm is defined as the ratio LZ / MZ.

For future purposes, two circles may be constructed, as shown, centred upon Z and of radii LZ and MZ. These will be referred to as **'torque arm circles'**, for obvious reasons.

EXIT GEOMETRY

In Figure 7.32, the line DF represents the most direct connection between the exit pallet locking corner at the start of impulse (immediately upon capture) and the corresponding position of the exit pallet arm pivot pin.

HZ is the torque arm of the exit pallet arm, DF, at the start of impulse (immediately upon capture).

EZ is the torque arm of the exit pallet arm, CG, at the end of impulse (at the time of release).

The torque arm ratio for the exit pallet arm is defined as the ratio HZ / EZ.

Torque arm circles are constructed, centred upon Z, of radii HZ and EZ.

CSM STIPULATIONS

CSM Stipulation 2 requires that the mean torque arm be 1/100th of the effective pendulum length. The mean torque arm is the mean (average) of LZ, MZ, HZ and EZ. A seconds beating pendulum will typically be 994.156mm in length (if located in London, England), for which the mean torque arm as 9.94156mm. Stipulation 2 may be easily achieved by simultaneously enlarging or reducing the entire escapement, including the escape wheel and the separation of the escapement frame arbor and the escape wheel arbor.

CSM Stipulation 3 and our described approach require that the torque ratio for the entry and exit sides of the escapement must each be 2 / 3. If it is assumed that the force along each pallet arm is constant, we may equate the mean torque ratio to the mean torque ***arm*** ratio, thereby avoiding unwieldy complication.

The effect of variations in force along each pallet arm (as the arm deviates from being tangential to the escape wheel) may be incorporated at a later stage, although the sharing of common torque arm circles prevents complete compensation (fortunately, the error is extremely small).

The mean torque arm ratio is the mean (average) of LZ / MZ and HZ / EZ, which is quite straightforward to determine. Achieving Stipulation 3 on that basis is, however, an extremely involved task, requiring detailed analysis and explanation. Two methods will be presented, in separate sections. The first method, graphical, will serve, if nothing else, to identify the effects of altering significant parameters. The second method, mathematical, will significantly ease the design process and provide valuable precision.

GRAPHICAL DESIGN PROCESS

Graphical (drawing) design methods, however carefully and correctly performed, will inevitably introduce inaccuracies, both in terms of drawing and of measuring. The geometry of the twin pivot grasshopper does, admittedly, tend to encourage such errors, as a result of a marked sensitivity to small movements in some areas and during certain phases of operation. In addition, as will soon be appreciated, the graphical methods presented herein can be extremely tedious, in that a degree of trial-and-error is required, albeit minimised by adopting an intelligent approach.

It will be assumed that the chosen design constraints comply with Harrison's CSM stipulations. Instructions are also provided to enable the incorporation of any desired set of constraints.

The graphical method suggested herein is based on the valuable condition that both the entry pallet and the exit pallet will independently achieve a torque arm ratio, from start to end of impulse, of precisely 2 / 3. As will become apparent, the incorporation of equal length pallet arms (from locking corner to pivot), although not impossible, would require considerable additional effort. However, it is all too easy to lose sight of the origins of equal pallet arms; they were incorporated entirely as a means of easing the *mathematical* procedure. As such, there is no justification for their inclusion in the *graphical* design process, since they add to, rather than reduce, the difficulty of the task (note, however, that the graphical design process will be illustrated shortly, using a geometry derived, for convenience and precision, from the mathematical model; equal length pallet arms will, therefore, be illustrated).

Apart from the undoubtedly beautiful and fascinating nature of the subject, the presentation of a graphical design method serves to clearly demonstrate how stipulations may be met and which designer chosen parameters affect which final outcomes. Any designer attempting to create a twin pivot grasshopper escapement, by whatever method, should be familiar with those causes and effects if they wish to avoid an unnecessarily lengthy, disorganised and frustrating process. The twin pivot geometry can be considerably less cooperative than the single pivot in normal circumstances; it would be foolhardy to permit ignorance to add to the difficulty.

Figures 7.29 to 7.36 and the accompanying instructions and comments explain the graphical design process in detail. The figures will be presented to a scale best suited to the layout. When creating new escapement geometries, designers are advised to repeat the sequence presented herein to as large a scale as available drawing equipment will permit, in order to minimise drawing and measurement inaccuracies. For the same reasons, it is suggested that, unless a large and accurate protractor is available, angles should be produced by resolving them, using the trigonometric function 'tan', into the two shortest sides of the largest possible right-angled triangles. Accuracy may thus be enhanced by laying out those sides vertically and horizontally, as appropriate.

Each significant stage in the graphical construction is presented in a progressive sequence of figures. For colour publications, as far as possible, **green** will represent escapement entry constructions and **red** will represent exit constructions. As each stage in the sequence progresses, previously green or red coloured elements will change to **black**, in order to indicate that they are no longer current constructions, but are carried over from a previous stage in the sequence. Newly added elements will continue to observe the green and red convention. When new elements are neither entry or exit orientated, they will be in **blue**, in order to highlight their introduction or use. This method of presentation will become perfectly clear as the sequence unfolds.

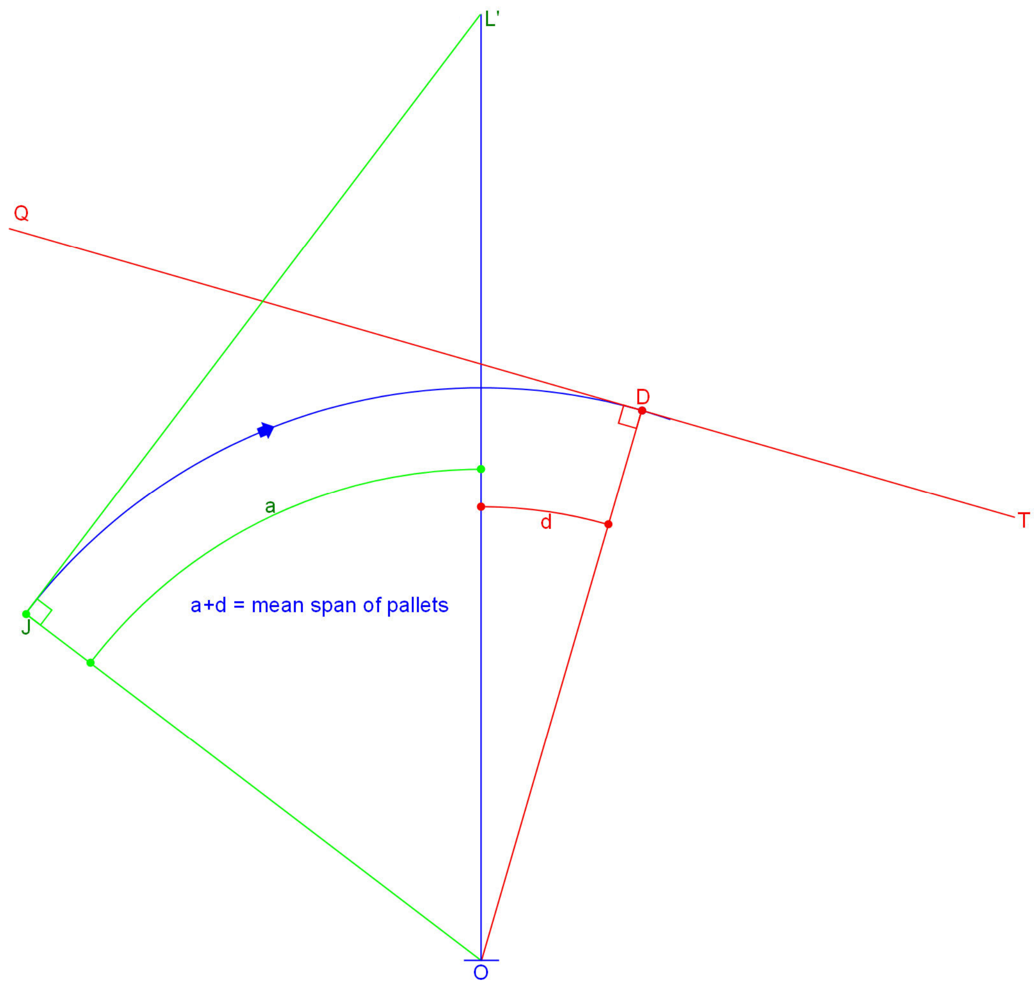


Figure 7.29 - Graphical Construction - Step 1

STEP ONE

Mark a point, O, at the bottom of the page, approximately central to the vertical page edges. Draw a vertical line through point O, as long as the page will allow.

Draw an arc, centre at O, representing the pitch circle of the escape wheel teeth tips. The radius may be arbitrary.

Choose (estimate) angle d, clockwise from the vertical through O. Draw OD. Early estimates of angle 'd' will probably require adjustment later. The proportions illustrated offer sensible (albeit, at this stage, unavoidably imprecise) guidance.

Choose the mean number of escape wheels teeth to be spanned by the pallets (whole number of tooth spaces, plus half a tooth space). Convert to an angle, in degrees. That angle is 'a + d'.

Draw OJ, anticlockwise removed from OD by angle 'a + d'

Draw JL', perpendicular to OJ. This is the direction of the applied force, in compression, produced by the entry pallet arm at the start of impulse.

Draw TQ, perpendicular to OD. This is the direction of the applied force, in compression, produced by the exit pallet arm at the start of impulse.

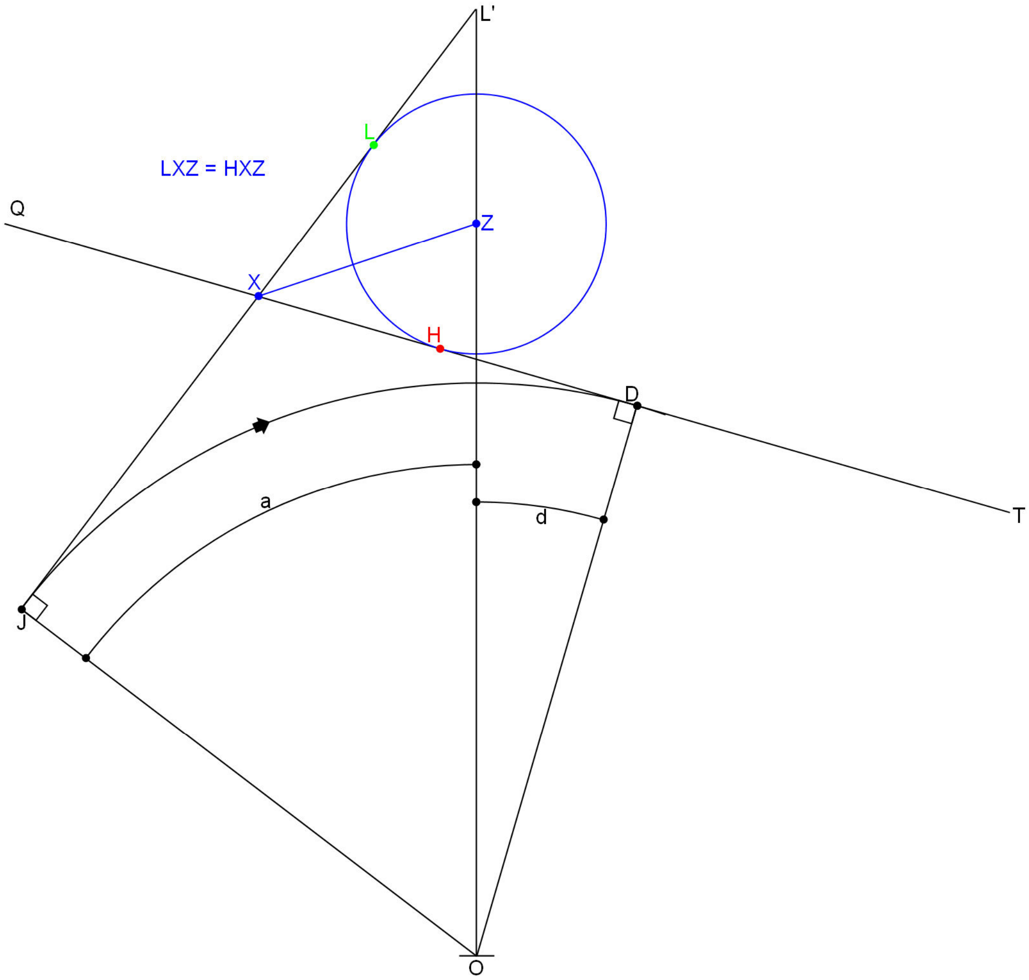


Figure 7.30 - Graphical Construction - Step 2

STEP TWO

Point X is established where TQ intersects JL'.

Construct XZ, bisecting angle DXL'.

Construct a circle, centre Z, tangential to both JL' and DQ. This is the smaller torque arm circle, of radius two units (being the 2 of the 2/3 rds torque ratio of Harrison Stipulation 3). The entry and exit pallet thrust lines at the start of impulse (JL and DH respectively) are tangential to this circle, representing a torque arm about Z of two units in both cases.

Note: Any desired torque ratio may be incorporated. At this stage, the torque circle will represent the numerator of a torque ratio defined as a fraction, or the ratio itself when that ratio is expressed a single, decimal number.

Thus, if the torque ratio, 't' equals 'r' divided by 's', the torque circle constructed in this step will represent 'r'.

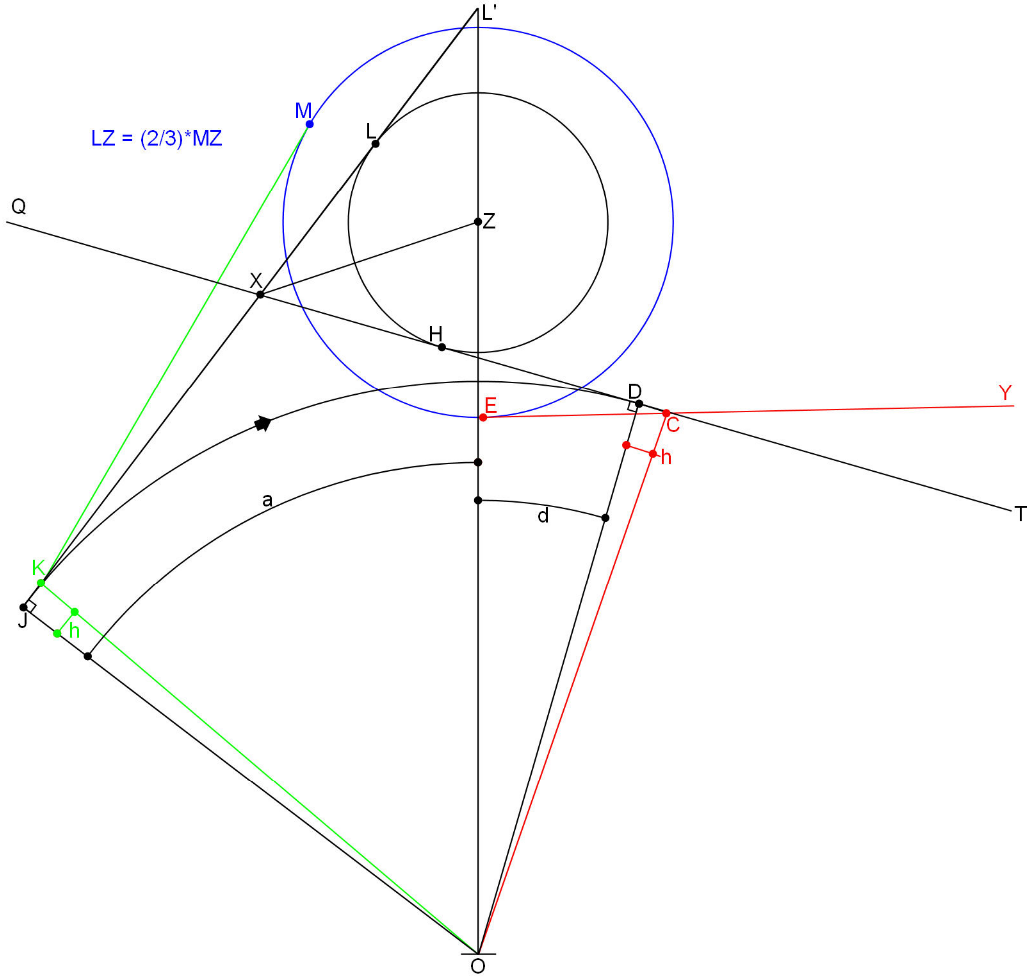


Figure 7.31 - Graphical Construction - Step 3

STEP THREE

Construct a circle, centre Z, of radius 3/2 of the radius of the previously drawn circle passing through L and H. Alternatively expressed, this is the larger torque arm circle, of radius three units (being the 3 of the 2/3 rds torque ratio of Harrison Stipulation 3).

Construct lines OK and OC, half an escape wheel tooth space (angle h) clockwise removed from J and D respectively. K represents the position of the entry pallet locking corner at the end of impulse and C represents the position of the exit pallet locking corner at the end of impulse.

Draw a line from K, tangential to the torque arm circle of radius three units. That line, KM, is the thrust line, in compression, of the entry pallet at the end of impulse.

Draw line from C, tangential to the torque arm circle of radius three units. That line, CE, is the thrust line, in compression, of the exit pallet at the end of impulse. It will serve a future purpose if CE is extended to Y.

Note: Any desired torque ratio may be incorporated. At this stage, the torque circle will represent the denominator of a torque ratio defined as a fraction, or unity when the torque ratio is expressed a decimal number.

Thus, if the torque ratio, 't' equals 'r' divided by 's', the torque circle just constructed will represent 's'.

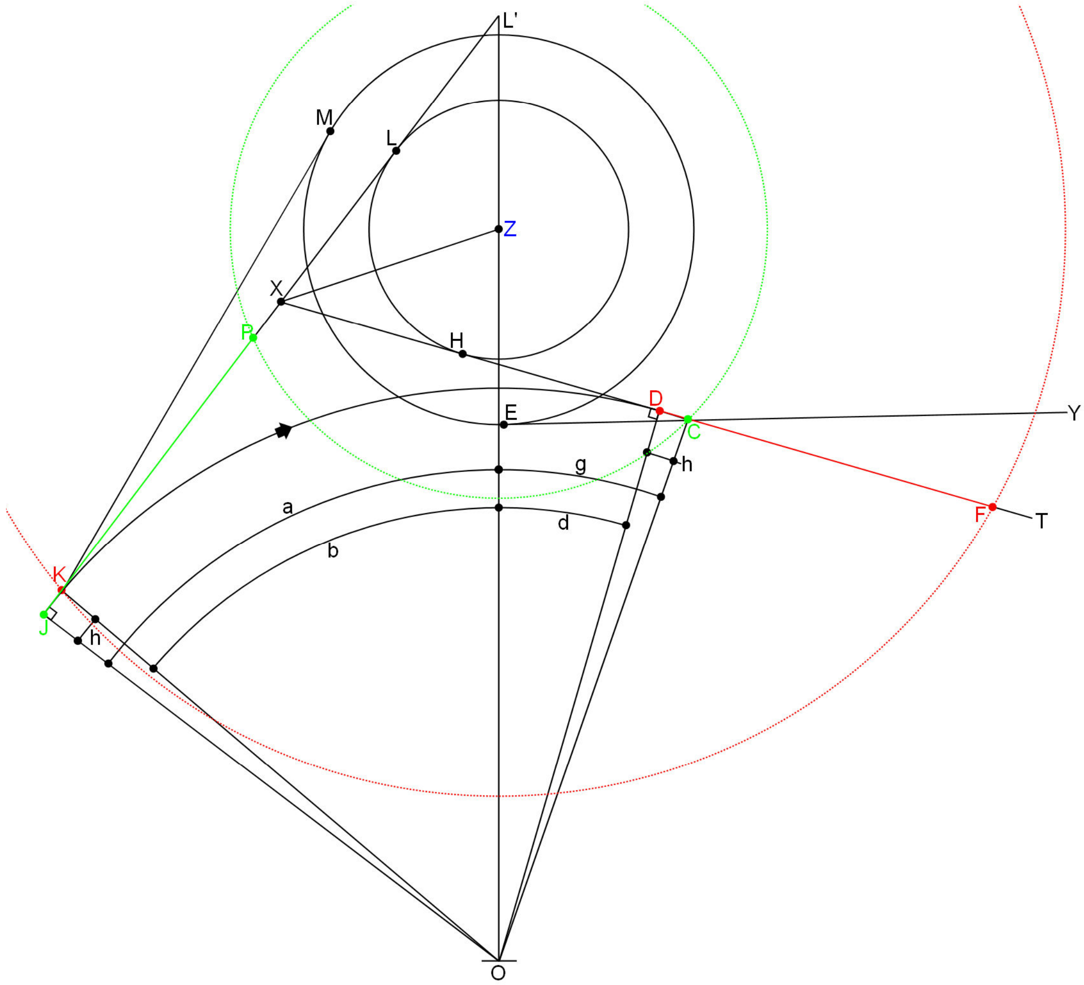


Figure 7.32 - Graphical Construction - Step 4.

STEP FOUR.

Draw "guide circle", centre Z, radius CZ. The circle intersects JL' at point P. JP is a first estimate of the distance from the entry pallet locking corner, J, to the entry pallet pivot, P, at the start of impulse.

Draw "guide circle", centre Z, radius KZ. The circle intersects TX at point F. DF is a first estimate of the distance from the exit pallet locking corner, D, to the exit pallet pivot, F, at the start of impulse.

NB - Guide circles may not offer useful assistance should the designer choose to deviate from Harrison's Stipulation 3 (i.e. a torque ratio of two to three).

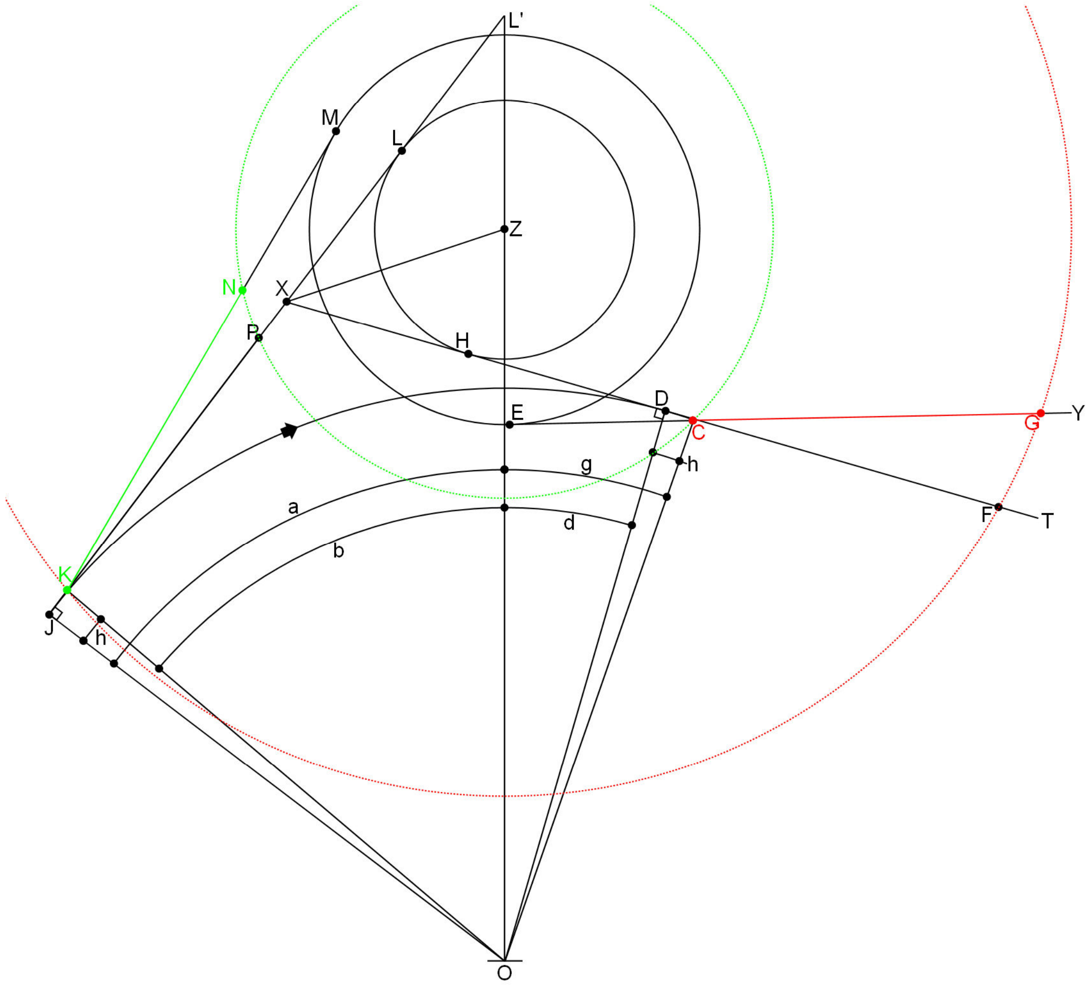


Figure 7.33 - Graphical Construction - Step 5

STEP FIVE

"Guide circle", centre Z, radius CZ (constructed in STEP FOUR) intersects KM at N. KN is a first estimate of the distance from the entry pallet locking corner to the entry pallet pivot at the end of impulse.

"Guide circle", centre Z, radius KZ (constructed in STEP FOUR) intersects CY at G. CG is a first estimate of the distance from the exit pallet locking corner to the exit pallet pivot at the end of impulse.

Determine if KN and JP are equal, within the range of drawing error and that DF and CG are equal, within the range of drawing error (note, in passing, that the guide circles automatically ensure that NZ=PZ and FZ=GZ).

If equality cannot be achieved in either, or both cases (i.e. KN doesn't equal JP and/or DF doesn't equal CG), then one or both pallet lengths at the start of impulse (i.e. JP and/or DF) must be altered and the process repeated from the beginning of step 4. However, the guide circles (if still considered useful) must be drawn through the revised point(s) P and/or F, *not* through C and/or J.

NB - Guide circles may not offer useful assistance should the designer choose to deviate from Harrison's Stipulation 3 (i.e. a torque ratio of two to three).

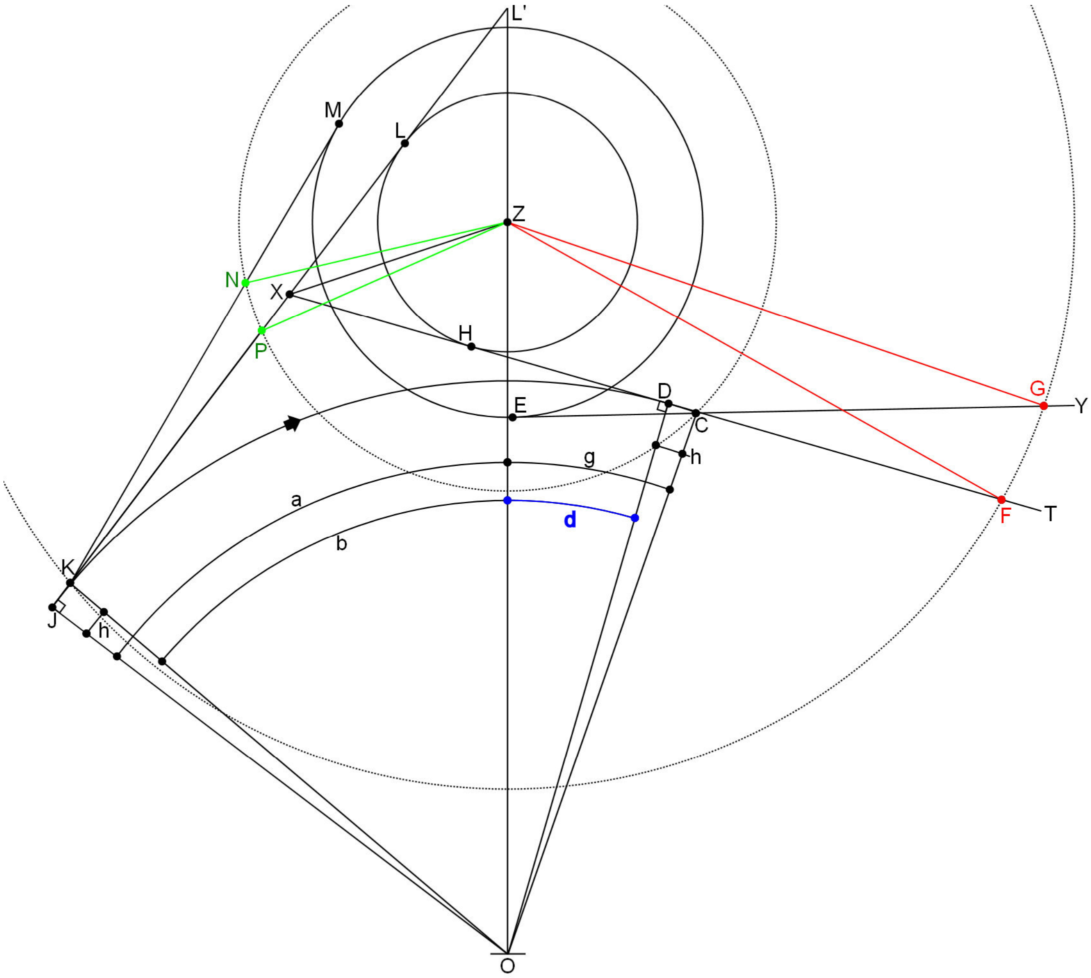


Figure 7.34 - Graphical Design - Step 6

STEP SIX

Construct lines NZ and PZ. Angle NZP represents the pendulum arc generated by the entry pallet.

Construct lines FZ and GZ. Angle FZG represents the pendulum arc generated by the exit pallet.

The two pendulum arcs NZP and FZG must be equal. For this example, eleven degrees is the assumed nominal target pendulum arc.

If the pendulum arcs are not equal, alter angle 'd' and repeat the entire process from the beginning of STEP ONE. It will speed the process of iteration if some intelligence is applied, by recording angle 'd' versus angles NZP and FZG. Over a series of adjustments, trends may thus be established, enabling appropriate adjustments, in magnitude and sense, to be made. If no solution can be found, then the number of teeth spanned by the escapement, represented by angle a + d, must be altered and the whole process repeated from the beginning of STEP ONE. Having established a viable solution, the resulting pendulum arc itself must be assessed. If the pendulum arc is not as desired, alter the total number of escape wheel teeth, N and repeat the entire process from the beginning of STEP ONE.

NB - for improved precision, account should be taken of the variation in impulse forces along each pallet arm. METHOD - Measure angles MKO and GCO. Draw two right angled triangles, one containing angle MKO-90, the other containing angle GCO-90. The length of the longest side (hypotenuse) represents the force at the start of impulse 'Fs'. The length of the slightly shorter side represents the force at the end of impulse 'Fe'. Start again from STEP ONE, incorporating the initial torque ratios multiplied by the ***mean*** of Fe/Fs (entry) and Fe/Fs (exit). The mean of the ratios must be used, because both the entry and exit sides of the geometry must continue to share only two common torque circles. Such an approach fails to achieve perfect compensation, but is the only viable method.

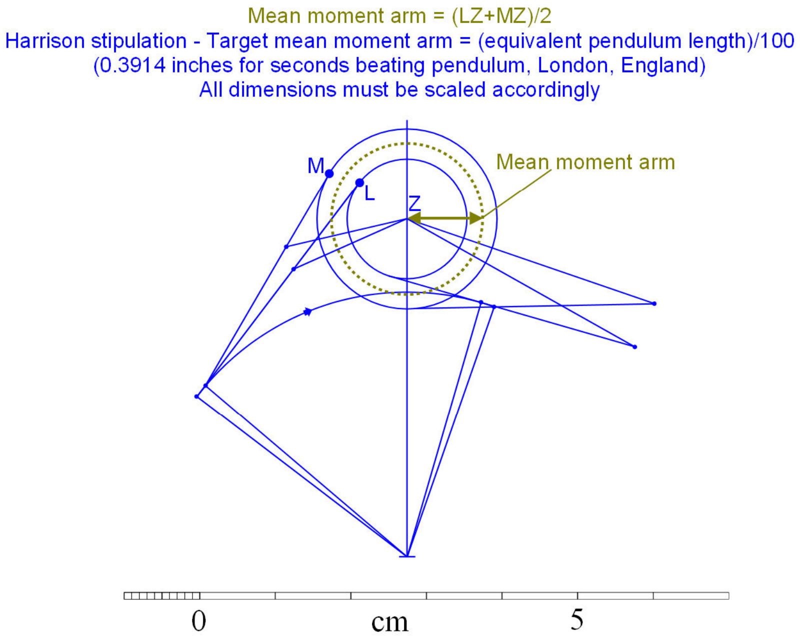


Figure 7.35 - Graphical Design - Step 7.

STEP SEVEN

Calculate the mean moment arm (LZ + MZ) / 2. This should equal one hundredth of the equivalent (idealised) pendulum length.

Note that the equivalent pendulum length is not the overall physical length of the intended pendulum, but is a theoretical length. It is the distance from the point of an idealised suspension to the centre of gravity of a bob at which the entire mass of the pendulum is concentrated, as explained in detail in PART TWO for the single pivot grasshopper escapement.

For London, England, it may be assume that a seconds beating pendulum will have an equivalent pendulum length of 994.156mm (39.14 inches)

The required mean moment arm of the escapement should, therefore, be 994.156/100 = 9.94156mm (0.3914 inches).

If that is not the case for the graphical construction thus far (as is likely), multiply every linear dimension by the ratio of the required mean moment arm to the drawn mean moment arm. Angles, however, must not be altered. The result is illustrated above.

Any desired ratio of equivalent pendulum length to mean moment arm may, obviously, be incorporated.

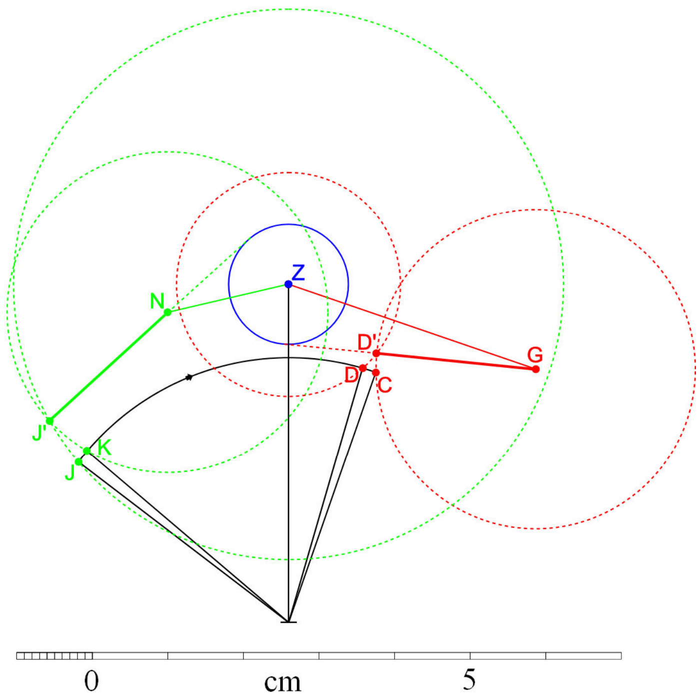


Figure 7.36 - Graphical Design - Step 8.

STEP EIGHT

Draw a circle, centre Z, passing through J. Draw a circle, centre N, passing through K. The two circles intersect at J' (and at one other point, further from the escape wheel, which may be ignored). J' represents the position of the entry pallet locking corner after release and upon pallet arm contact with the entry composer (ignoring time in transit between entry pallet nib release and composer contact). KJ' represents the 'lift' of the entry pallet upon release.

Draw a circle, centre Z, passing through D. Draw a circle, centre G, passing through C. The two circles intersect at D' (and at one other point, further from the escape wheel, which may be ignored). D' represents the position of the exit pallet locking corner after release and upon pallet arm contact with the exit composer (ignoring time in transit between exit pallet nib release and composer contact). CD' represents the 'lift' of the exit pallet upon release.

Obviously, once the construction is understood, it will be unnecessary to draw entire circles.

There is an assumption that the time interval between pallet release and resting upon the composer occupies no rotation of the escapement frame arbor. The main purpose of these constructions is to check that pallet nib lift is sufficient and as required. The construction will also enable a determination of the maximum nib length, ignoring additional clearance demands. Acceptable minimum pallet nib lift obviously depends upon the likely quality of construction and the intended operating environment.

If no satisfactory solution can be found, then the total number of escape wheel teeth must be altered and the whole process repeated from STEP ONE.

It will be instructive to analyse the positions of the pallet arms at the instant they come to rest upon their paired composer, after release from the escape wheel (i.e. points J' and D'). For both the entry and exit pallet arms, the extensions of lines joining the locking corners to the pallet arm pivots (shown above as broken lines) will be tangential to the inner torque arm circle (of two units radius). That condition prevails until the relevant pallet locking corner is captured by an escape wheel tooth tip and resumes after pallet release.

GRAPHICAL DESIGN PROCESS
SUMMARY

Torque ratios are incorporated without any requirement for iteration. They are simply drawn to the required proportions and the remainder of the geometry must conform.

(1) - Entry pallet arm length (locking corner to pivot) must be altered until it is the same for both the start and the end of entry pallet impulse (i.e. JP equals KN). Exit pallet length must be altered until it is the same for both the start and the end of exit pallet impulse (i.e. DF equals CG). Achieving equal length exit and entry pallet arms is possible, but *extremely* tedious, completely unnecessary and totally inappropriate to the graphical design method.

(2) - The value of angle 'd' must be adjusted until the pendulum arc generated by the entry pallet equals that for the exit pallet.

(3) - Having achieved (2), the mean number of teeth spanned (represented by angle 'a + d') must be altered in order to adjust the magnitude of the pendulum arc (if necessary).

(4) - Should a solution prove to be impossible, or not as desired, the total number of escape wheel teeth must be adjusted and the entire process repeated from the beginning.

(5) - Having established a compliant geometry, the entire escape wheel and escapement geometry must be scaled, in order that the mean torque arm is the required proportion of the equivalent pendulum length (being 1/100 for compliance with Harrison's Stipulation 2).

GRAPHICAL DESIGN PROCESS
CONCLUSIONS

It must now be obvious that the graphical design method has the potential to be *exceptionally* tedious, certainly more so than for the single pivot escapement. Although the process is certainly not random (provided that intelligent trend recognition is employed, as advised) it is obviously based upon an extremely time consuming, iterative process. Whilst being an inconvenience, rather than an insurmountable obstacle, when such difficulties are combined with the disadvantages of drawing and measurement inaccuracies and a geometry markedly sensitive to such, the conclusion must be that a speedier and more precise design process is required.

MATHEMATICAL DESIGN PROCESS

For the benefit of those who are averse to mathematics, this chapter may be completely ignored without detriment to a general understanding of the grasshopper escapement.

MATHEMATICAL DESIGN PROCESS
OBJECTIVE AND BASES OF ANALYSIS

The objective is to produce a universally applicable mathematical method of designing the twin pivot grasshopper escapement. To that end, those parameters that uniquely define the escapement must be identified. Parameters will require separation into those that must be chosen by the designer and those to be be generated by mathematical manipulation.

DESIGNER CHOSEN	MATHEMATICALLY GENERATED
Desired nominal pendulum arc, p	Achieved pendulum arc, p
Initial escape wheel tooth tips radius, R	Final escape wheel tooth tips radius, R*
Total number of escape wheel teeth, N	Escapement frame arbor to entry pallet, PZ
Mean number of escape wheel teeth spanned, n	Escapement frame arbor to exit pallet, FZ
Angle 'd' (see geometry)	Entry pallet locking corner lift upon release, J'K
Desired entry pallet locking corner to pivot, JP	Exit pallet locking corner lift upon release, CD'
Desired exit pallet locking corner to pivot, DF	Escape arbor to escapement frame arbor, OZ
Equivalent pendulum length, L*	

Figure 7.37 (next) shows the twin pivot grasshopper escapement geometry, without torque arm circles, split along a line through the escape wheel and escapement frame arbors into two separate entities, one for the entry pallet geometry, to the left of the split and one for the exit pallet geometry, to the right. For publications in colour, green will continue to indicate elements for the entry side of the escapement and red will indicate those for the exit side. Figure 7.37 serves to illustrate that there are only four naturally occurring, defining links between the entry and exit sides. The first two links are a sharing of the escape wheel and escapement frame arbors, which, in isolation, impose the somewhat limited (albeit essential) constraints of escape wheel tooth tip loci and a common dimension, OZ. The third and fourth links, that the entry and exit sides must generate the same pendulum arcs, are certainly two constraint of considerable value. Unfortunately, an attempted analysis based upon those constraints alone confirms that a meaningful solution is impossible.

However, if torque arm circles are included, as shown in **Figure 7.38 (next but one)**, then two more links and a solution emerge quite naturally. Provided that the entry and exit geometries are constructed around equal diameter torque arm circles at the start of impulse and equal diameter torque arm circles at the end of impulse, an invaluable link between the entry and exit sides of the escapement is created. The sharing of common torque arm circles (which, for Harrison's CSM stipulations, are of two units radius at the start of impulse and three units radius at the end of impulse) obliges both sides of the escapement to be compatible throughout the entire cycle of operation, not just at the start and end of impulse. This is the indispensable basis of a successful mathematical analysis.

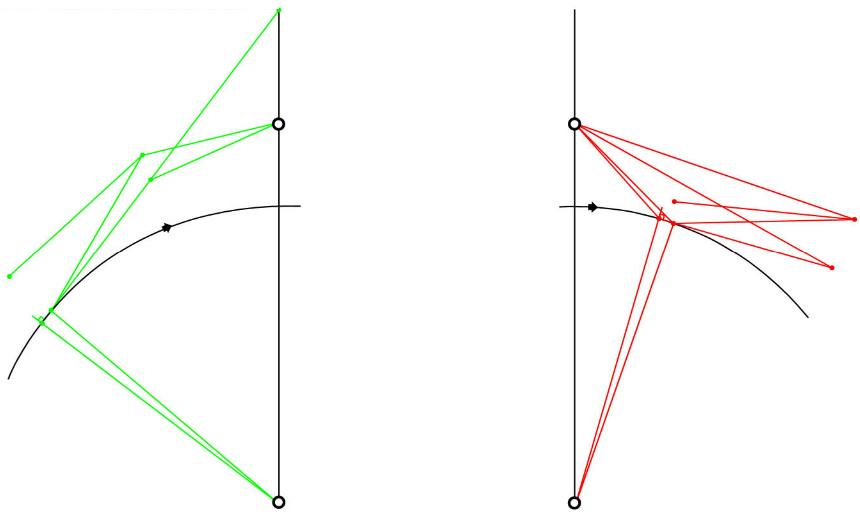


Figure 7.37 - Entry and exit geometries independent, except at shared arbors.

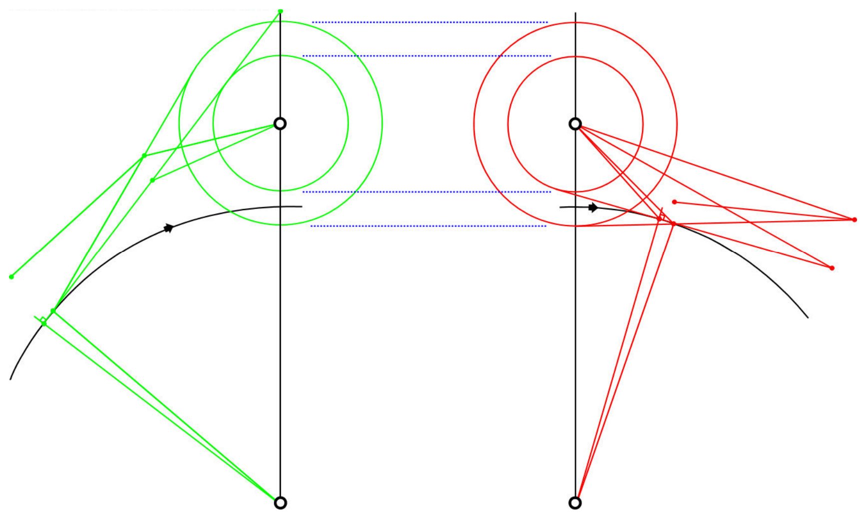


Figure 7.38 - Entry and exit geometries unified by common torque arm circles.

The analysis may be forced to comply with the Harrison stipulation that the mean torque arm must be one hundredth of the length of the idealised equivalent pendulum, by virtue of simple scaling of all linear dimensions (*not* angles). Whilst any equivalent pendulum length may be chosen, Harrison stipulated that a 'long pendulum' should be used with his single pivot escapement, indicating that a seconds beating pendulum would be acceptable.

Observe that the line of force along each pallet arm at the start of impulse is tangential to the escape wheel at the location of the engaged tooth tip. Common sense and Harrison's layout drawing, MS 3972/3 for his single pivot grasshopper escapement are the bases of this decision.

The mathematical method initially assumes that the force along each pallet arm remains constant during the escapement cycle. That is not strictly the case, by virtue of the component of escape wheel delivered impulse along each arm suffering alteration as the arm rotates about the pallet pivot and the escapement frame arbor. The inintial approximation permits an assumption that the mean torque ratio may be equated to the mean torque ***arm*** ratio. The alterations in forces along the pallet arms during impulse are incorporated at a later stage, although that correction may be omitted, if so desired, with only minor consequences.

MATHEMATICAL MODEL

Figure 7.39 (next) and the applicable guide (overleaf) identify the points and angles used throughout the mathematical modelling process. It will be instructive to note that Figure 7.39 is derived from Figure 7.31, with additional points and lines. The purpose of those additions is to enable a conversion of the geometry to the necessary number of connected right-angled triangles, in order that a mathematical analysis may be performed. For publications in colour, green, red and blue are used to create a geometry that may be interpreted both quickly and easily. Green and red represent entry and exit constructions, respectively. Blue is used, in this particular instance, as a means of identifying geometries at the start of impulse, on both the entry and exit sides.

Figure 7.40 (next but one) presents a magnified view of the geometry, close to and below the escapement frame arbor, where some potentially confusing congestion occurs. Some lettering has been modified and/or relocated slightly in order to improve clarity at the increased scale. Particular care must be taken to distinguish between four potentially misleadingly constructions. The first two constructions are right-angled triangles ADZ and BCZ, which lie along and perpendicular to line OZ. The third construction is the right-angled triangle, CEZ. Side EZ does ***not*** lie along OZ (except, perhaps, by rare coincidence). Side CE is tangential, at E, to the larger torque arm circle. The fourth construction is right-angled triangle DHZ, for which DH (<u>not</u> CH) is tangential to the smaller torque arm circle.

Calculations, listed in a logical sequence, will be presented such that the purpose and derivation of every equation is absolutely clear. To that end, in addition to the normal flow of equations from top to bottom of the page, they will also progress from left to right, when relevant. Thus, an essential parameter will be identified and its most obvious method of calculation listed in what might be termed a "primary" equation to the extreme left of the page. The subsequent determination of any unknown parameters within that primary equation will generate a requirement for "secondary" equation(s), which will be listed below and immediately to the right of the primary equation. Unknown parameters within the secondary equation(s) will require "tertiary" equations, listed below and immediately to the right of the secondary equation. Equations with more than one unknown parameter will require as many sets of subsidiary equations. This process continues until all relevant parameters have been determined.

The following will emphasise the layout:-

PRIMARY EQUATION
    SECONDARY EQUATION
        TERTIARY EQUATION
    SECONDARY EQUATION
        TERTIARY EQUATION
            ETC........(AS REQUIRED)

It is essential to note that, once a parameter has been determined, its derivation will not be repeated should it arise in subsequent calculations. Thus, if a parameter is apparently bereft of a source equation, this should be taken as a cue to review preceding calculations in search of the earlier derivation.

An obvious feature of the mathematical modelling technique is that, beyond the more obvious limits of physical possibility, impracticable designs can be generated, without necessarily being apparent. Neatly printed, computer generated figures do not guarantee correct information. In an effort to avoid damaging errors, the numerical results of escapement calculations should be physically drawn, to as large a scale as available drawing equipment will permit. That precaution will confirm, within the limits of drawing accuracy, that the escapement design is practicable and that stipulations have been met.

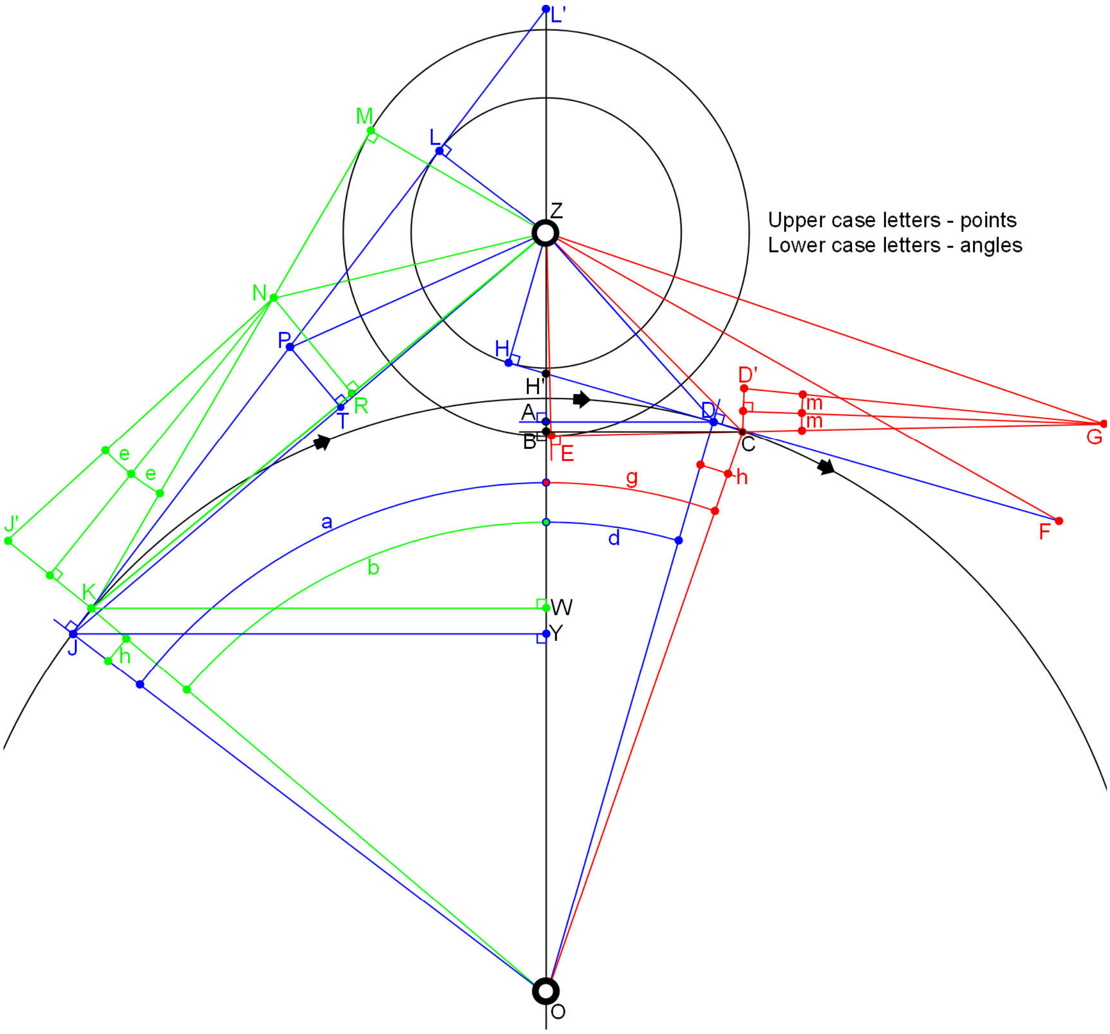


Figure 7.39 - Annotated geometry for the mathematical model.

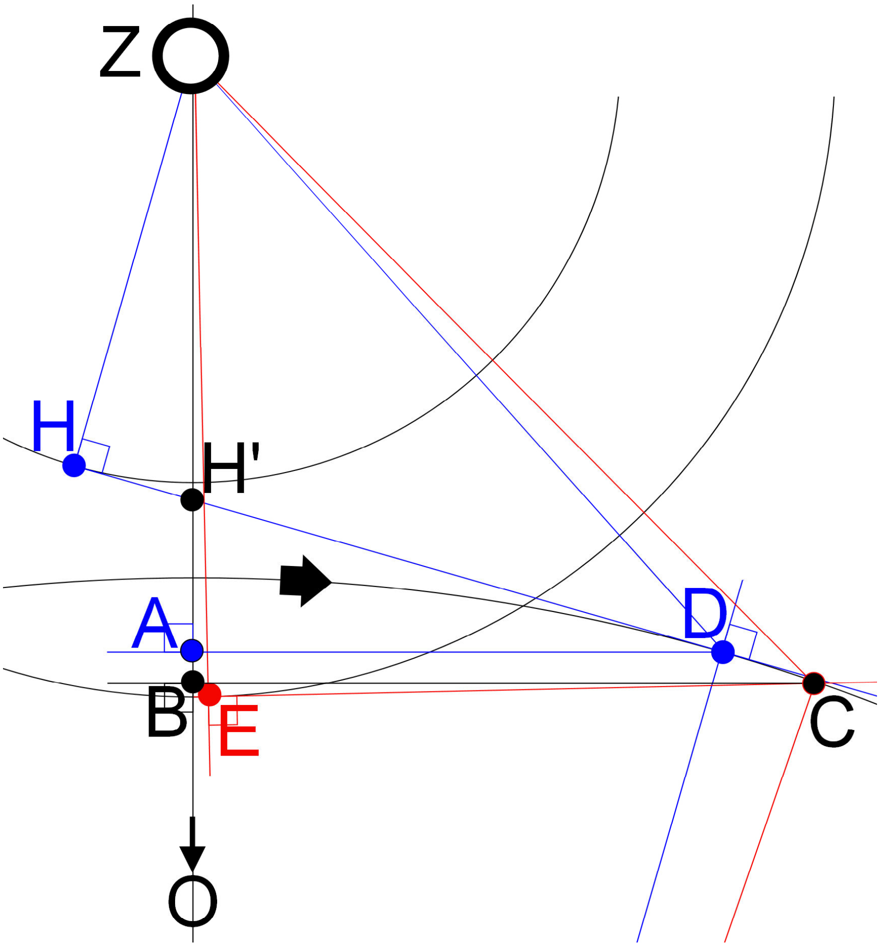


Figure 7.40 - Magnified view of part of Figure 7.39

GUIDE TO FIG. 7.39 AND (IN PART) FIG. 7.40
(in alphabetical order)

Arc through J, K, D and C - locus of escape wheel tooth tips.

a +d - The 'span' of the escapement, in degrees.

C - Exit pallet locking corner at the start of release (end of impulse).

CD' - Exit pallet travel ('lift') during release.

d - designer chosen angle.

D - Exit pallet locking corner upon capture (start of impulse). DF is perpendicular to DO.

D' - Exit pallet locking corner at the end of release (resting upon exit composer).

DF - exit pallet pivot to locking corner distance - designer chosen.

e - During entry pallet nib lift, pallet arm rotates through angle 2e.

EZ - Exit pallet torque arm at the end of impulse.

F and G - Exit pallet pivot at start and end of impulse, respectively.

h - The angle subtended at O by half a tooth space.

h - half the angle subtended at O by two adjacent escape wheel tooth tips.

HZ - Exit pallet torque arm at the start of impulse.

J - Entry pallet locking corner upon capture (start of impulse). JP is perpendicular to JO.

J' - Entry pallet locking corner at the end of release (resting upon entry composer).

JP - entry pallet pivot to locking corner distance - designer chosen.

K - Entry pallet locking corner at the start of release (end of impulse).

KJ' - Entry pallet travel ('lift') during release.

L* - equivalent pendulum length (idealised pendulum) **-** designer chosen. Harrison specifies a 'long pendulum'.

LZ - Entry pallet torque arm at the start of impulse.

m - During exit pallet nib lift, pallet arm rotates through angle 2m.

M - mean torque arm of escapement over one complete cycle of operation. Harrison specifies L*/100.

M* - mean torque arm of escapement over one complete cycle of operation, for escape wheel radius R*.

MZ - Entry pallet torque arm at the end of impulse.

n - mean number of escape wheel tooth spaces spanned by the pallet nib locking corners
(whole number, plus a half)

N - total number of escape wheel teeth **-** designer chosen (take care to distinguish from point N).

O - Escape wheel arbor. Fixed pivot (attached to an immovable object).

p - pendulum arc, from one extremity of swing to the other.

P and **N** - Entry pallet pivot at start and end of impulse, respectively.

R - chosen (arbitrary) escape wheel teeth tips pitch circle radius - designer chosen.

R* - escape wheel teeth tips pitch circle radius complying with Harrison stipulations, notably M* = L*/100.

t - torque arm ratio of the escapement over one complete cycle of operation.

T - one arbitrary unit of torque arm (total torque arms, start of impulse = 2T, end of impulse = 3T).

Z - Escapement frame arbor. Fixed pivot (attached to an immovable object).

MATHEMATICAL DESIGN PROCESS

MATHEMATICAL ANALYSIS

CALCULATION (1) - Complying with a chosen stipulation for mean torque arm ratio - t

Universal stipulation t = r / s (for Harrison's Stipulation 3, r = 2 and s = 3)

Let T be one unit of torque arm.

Entry pallet LZ = rT ------ **equation [1]**
MZ = sT ------ **equation [2]**

Exit pallet HZ = rT ------ **equation [3]**
EZ = sT ------ **equation [4]**

CALCULATION (2) - Matching OZ(entry) and OZ(exit) along OL'

OZ(entry) = L'O-L'Z
L'O = JO / cos(a)
JO = R
R = designer chosen
a = 2nh-d
n = designer chosen
h = 180 / N
N = designer chosen
d = designer chosen
L'Z = LZ / cosLZL'
LZL' = a
OZ(exit) = H'O+H'Z
H'O = DO / cos(d)
DO = R
H'Z = HZ / cosHZH'
HZH' = d

OZ(entry) = OZ(exit)
Thus R/cos(a)-LZ/cos(a) = R/cos(d)+HZ/cos(d)
or R/cos(a)-rT/cos(a) = R/cos(d)+rT/cos(d) **by applying equations [1] to [4]**

Thus T = R[1/cos(a)-1/cos(d)] / r[1/cos(a)+1/cos(d)] ------ **equation [5]**

LZ, MZ, HZ and EZ may now be determined, using equations [1] to [5]

CALCULATION (3) - Entry pallet escapement frame arbor to pallet pivot - PZ and NZ

PZ = sqrt(LP^2+LZ^2)
LP = JL-JP
JL = sqrt(JZ^2-LZ^2)
JZ = sqrt(YZ^2+JY^2)
YZ = OZ-OY
OY = JOcos(a)
JY = JOsin(a)
JP = designer chosen
NZ = sqrt(MN^2+MZ^2)
MN = KM-KN
KM = sqrt(KZ^2-MZ^2)
KZ = sqrt(WZ^2+KW^2)
WZ = OZ-OW
OW = KOcos(b)
KO = R
b = a-h
KW = KOsin(b)
KN = JP

CALCULATION (4) - Entry pallet generated pendulum arc - p'

p' = NZP
NZP = NZW-PZY
NZW = NZR+KZW
NZR = KZM-MZN
KZM = arctan(KM/MZ)
MZN = arctan(MN/MZ)
KZW = arctan(KW/WZ)
PZY = PZT+JZY
PZT = JZL-LZP
JZL = arctan(JL/LZ)
LZP = arctan(LP/LZ)
JZY = arctan(JY/YZ)

CALCULATION (5) - Exit pallet escapement frame arbor to pallet pivot - FZ and GZ

FZ = sqrt(FH^2+HZ^2)
FH = DH+DF
DH = sqrt(DZ^2-HZ^2)
DZ = sqrt(AZ^2+AD^2)
AZ = OZ-AO
AO = DOcos(d)
AD = DOsin(d)
DF = designer chosen
GZ = sqrt(EG^2+EZ^2)
EG = CE+CG
CE = sqrt(CZ^2-EZ^2)
CZ = sqrt(BZ^2+BC^2)
BZ = OZ-OB
OB = COcos(g)
CO = R
g = d+h
BC = COsin(g)
CG = DF

CALCULATION (6) - Exit pallet generated pendulum arc - p''

p'' = FZG
FZG = BZG-AZF
BZG = EZG+BZE
EZG = arctan(EG/EZ)
BZE = BZC-CZE
BZC = arctan(BC/BZ)
CZE = arctan(CE/EZ)

AZF = FZH-AZH
FZH = arctan(FH/HZ)
AZH = DZH-AZD
DZH = arctan(DH/HZ)
AZD = arctan(AD/AZ)

CALCULATION (7) - Accounting for the variation in force along each pallet arm during impulse.

s(new) = 2s(old) / {sinMKO+sinOCG}
MKO = MKZ+OKZ
MKZ = 90-KZM
OKZ = OKW+WKZ
OKW = 90-b
WKZ = 90-KZW
OCG = BZE+g+90

CALCULATION (8) - Complying with a chosen stipulation for mean moment arm - M*

R* = RM* / M
M* = L*/ y
L* = designer chosen
y = designer chosen (for Harrison's Stipulation 2, y = 100)
M = (rT+sT) / 2
Thus, from equation [5] ----- M = (r + s)R[{1/cos(a)-1/cos(d)} / {1/cos(a)+1/cos(d)}] / 2

CALCULATION (9) - Pallet nibs locking corner lifts immediately following release.

Entry pallet lift - J'K
J'K = 2KNsin(e)
e = 0.5(J'NZ-KNZ)
J'NZ = JPZ
JPZ = JPT+TPZ
JPT = 90-LJZ
LJZ = arctan(LZ / JL)
TPZ = 90-PZT
KNZ = KNR+RNZ
KNR = 90-MKZ
MKZ = arctan(MZ/KM)
RNZ = 90-NZR

Exit pallet lift - CD'
CD' = 2CGsin(m)
m = 0.5(CGZ-D'GZ)
CGZ = arctan(EZ/EG)
D'GZ = DFZ
DFZ = arctan(HZ/FH)

MATHEMATICAL DESIGN PROCESS
SUGGESTED SEQUENCE OF CALCULATIONS

(A) - Choose values for all designer-chosen parameters (d, JP (target), L*, n, N, p (target), R, t (= r/s, target)).

(B) - **See **.** Adjust JP until PZ = NZ, *then* determine pendulum arc, p' - See Calculations (3) and (4) respectively.

(C) - **See **.** Adjust DF until FZ = GZ, *then* determine pendulum arc, p'' - See Calculations (5) and (6) respectively.

****Experience reveals that computation is greatly eased by specifying that JP (and, therefore, KN) is equal to DF (and, therefore, CG).**

(D) - Repeat (B) and (C), noting previous calculations for trend guidance during iteration, until p' = p''

(E) - If a solution to (D) cannot be found, alter 'd' and repeat the analysis from (B).

(F) - If a solution to (E) cannot be found, alter 'n' and repeat the analysis from (B)

(G) - If a solution to (F) cannot be found, alter 'N' and repeat the analysis from (B)

(H) - To account for variation in force along each pallet arm during impulse (if desired), determine the modified value of parameter 's' - See Calculation (7). Repeat the entire analysis from (B).

(I) - Multiply all linear dimensions by R* / R - See Calculation (8)

(J) - Calculate pallet nib locking corner lifts following immediately release, based upon dimensions derived from (I) - See Calculation (9). If unacceptable, alter 'N' and repeat the entire analysis from (B)

(K) - Draw the derived geometry to as large a scale as available equipment permits, in order to identify any anomalies. Repeat the drawing to a scale of 1 to 1 and assess practicalities.

MATHEMATICAL DESIGN PROCESS
MATHEMATICAL MODEL OUTPUT

The mathematical model was used to produce a range of optimised grasshopper escapement geometries.

Harrison's stipulations are adhered to in all cases, unless stated otherwise.

The governing parameter is the total number of escape wheel teeth. In view of the common longcase precision regulator requirement to provide a seconds display at the escape wheel arbor, escape wheels of 30, 60, 90, 120, 150 and 180 teeth have been analysed. Other conclusions aside, an inspection of the geometries at each end of that range demonstrates that there would appear to be nothing to be gained, for the present purpose and in terms of conventional longcase regulator applications, from detailed studies of greater or lesser tooth counts.

Four objectives were established, as follows:

(1) - To identify the effect(s), if any, of escape wheel tooth count.

(2) - To determine maximum and minimum practicable escape wheel tooth counts, subject to sensible lower limits and bearing in mind the chosen application of longcase precision regulator .

(3) - To determine the optimum escape wheel tooth count (noting that this is *not* the same as optimising the geometry for a given tooth count).

(4) - To provide a range of optimised geometries, in order that designers or the mathematically averse would be relieved of the tasks of understanding the mathematical modelling process and/or of computation.

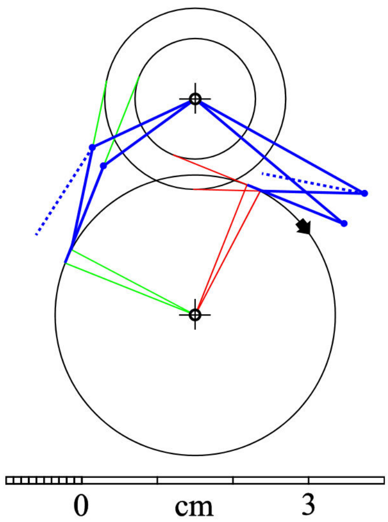
Torque arm ratios of two to three for both the entry and exit sides are incorporated.

Experience of using the mathematical model reveals that computation is greatly eased by specifying that JP (and, therefore, KN) is equal to DF (and, therefore, CG). In other words, the entry and exit pallet arm pivot to locking corner separations are made equal, purely in the interests of reducing workload (considerably). Should there be any requirement for differing pallet arms or a specific dimension, then such variations are obviously still available. There may be manufacturing advantages to be gained from identical pallet arms, although that is, perhaps, debatable.

A nominal pendulum arc of eleven degrees has been assumed. As a consequence of that additional constraint, a single solution emerges for each escape wheel tooth count. However, other strict geometrical constraints, notably the total number of escape wheel teeth and the mean pallets span, impose themselves rigidly, which demands some flexibility of the pendulum arc. As a result, the achieved arc inevitably deviates from the specified nominal figure.

Note that, in the interests of clarity and consistency, no account has been taken of the variation in force along each pallet arm during impulse. Designers wishing to incorporate that influence must perform the necessary calculations for themselves (see Calculation (7) of the mathematical model). The resultant maximum torque ratio error is estimated to vary from + 3.9 % (30 tooth escape wheel) to + 1.8 % (180 tooth escape wheel).

Each of the final designs is presented in tabular form, accompanied by a scale drawing of the geometry. This method of presentation will serve as a clear easy-reference for escapement designers and support eventual conclusions as to the optimum geometry. Numerical values are given to five decimal places, in order to assist those who wish to reproduce and/or confirm the calculations. The figures are presented with the escapement frame arbor and tables the in common positions on the page, which will enable speedy comparison, as required, albeit at the expense of neat appearances.



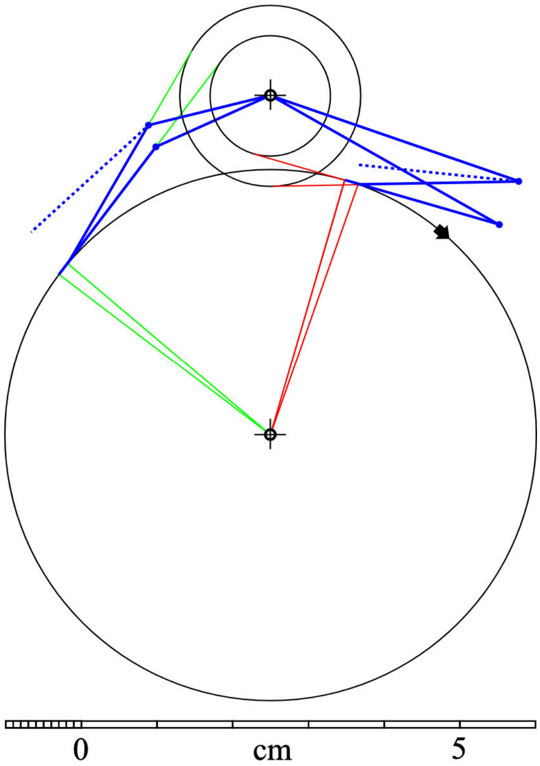
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 30
Escape wheel teeth tips pitch circle radius, R* mm = 18.51169
Mean number of teeth spanned, n = 7.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 21.75
Entry pallet locking corner to pivot, JP mm = 13.75988
Exit pallet locking corner to pivot, DF mm = 13.75988
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm = 28.49339
Escapement frame arbor to entry pallet-pivot, PZ mm = 14.98908
Escapement frame arbor to exit pallet-pivot, FZ mm = 25.58583
Entry pallet locking corner lift upon release, J'K mm = 4.94879
Exit pallet locking corner lift upon release, CD' mm = 2.32406
Pendulum arc, p degrees = 10.87573

FIGURE 7.40 - 30 TOOTH ESCAPE WHEEL



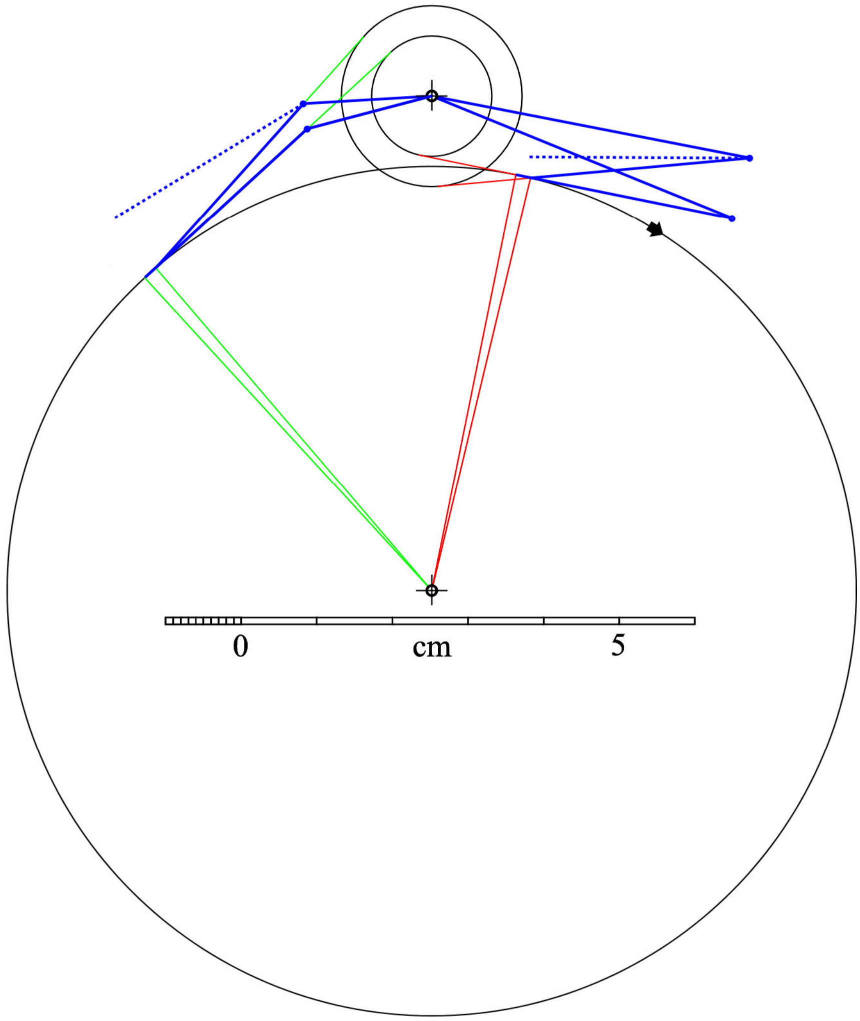
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 60
Escape wheel teeth tips pitch circle radius, R* mm = 35.11807
Mean number of teeth spanned, n = 11.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 16.262
Entry pallet locking corner to pivot, JP mm = 21.20251
Exit pallet locking corner to pivot, DF mm = 21.20251
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm = 44.86636
Escapement frame arbor to entry pallet pivot, PZ mm = 16.54280
Escapement frame arbor to exit pallet pivot, FZ mm = 34.69044
Entry pallet locking corner lift upon release, J'K mm = 6.41264
Exit pallet locking corner lift upon release, CD' mm = 2.53924
Pendulum arc, p degrees = 10.47295

FIGURE 7.41 - 60 TOOTH ESCAPE WHEEL



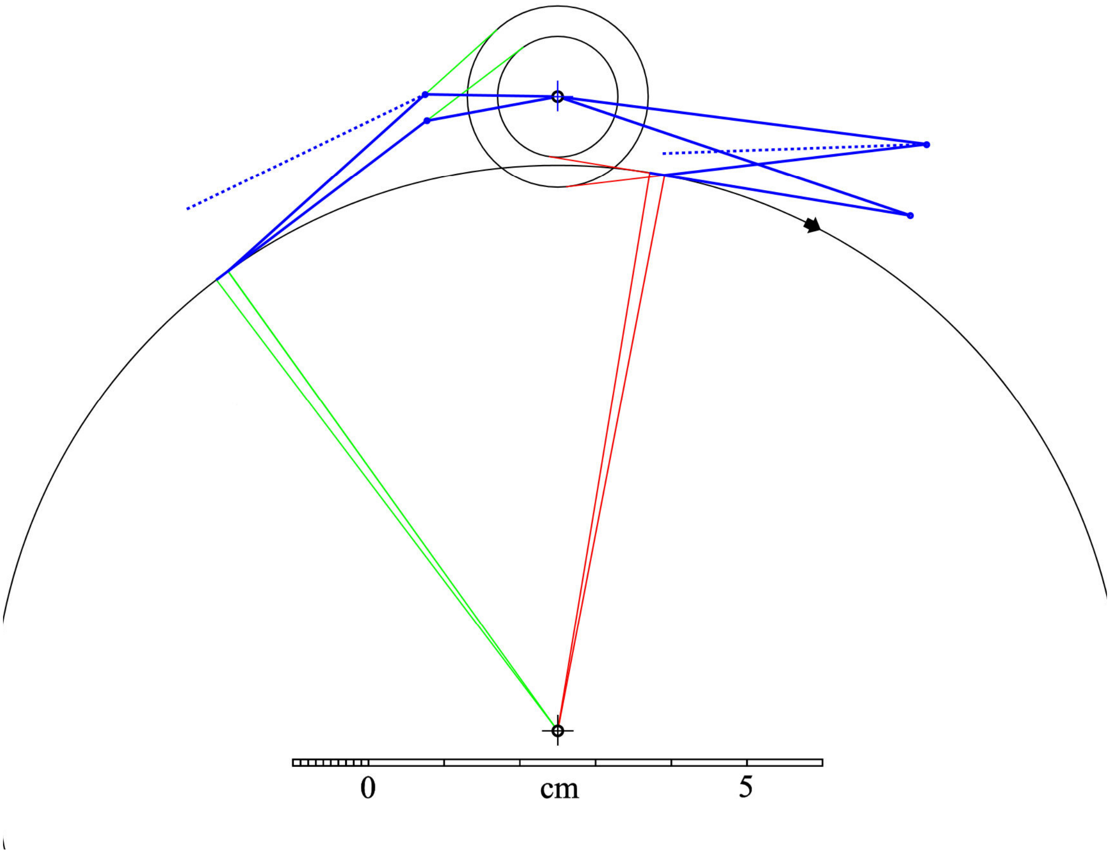
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 90
Escape wheel teeth tips pitch circle radius, R* mm = 56.13249
Mean number of teeth spanned, n = 13.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 11.46
Entry pallet locking corner to pivot, JP mm = 29.16434
Exit pallet locking corner to pivot, DF mm = 29.16434
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm =65.38937
Escapement frame arbor to entry pallet pivot, PZ mm = 17.01846
Escapement frame arbor to exit pallet pivot, FZ mm = 42.89982
Entry pallet locking corner lift upon release, J'K mm = 8.44686
Exit pallet locking corner lift upon release, CD' mm = 2.77954
Pendulum arc, p degrees = 11.15633

FIGURE 7.42 - 90 TOOTH ESCAPE WHEEL



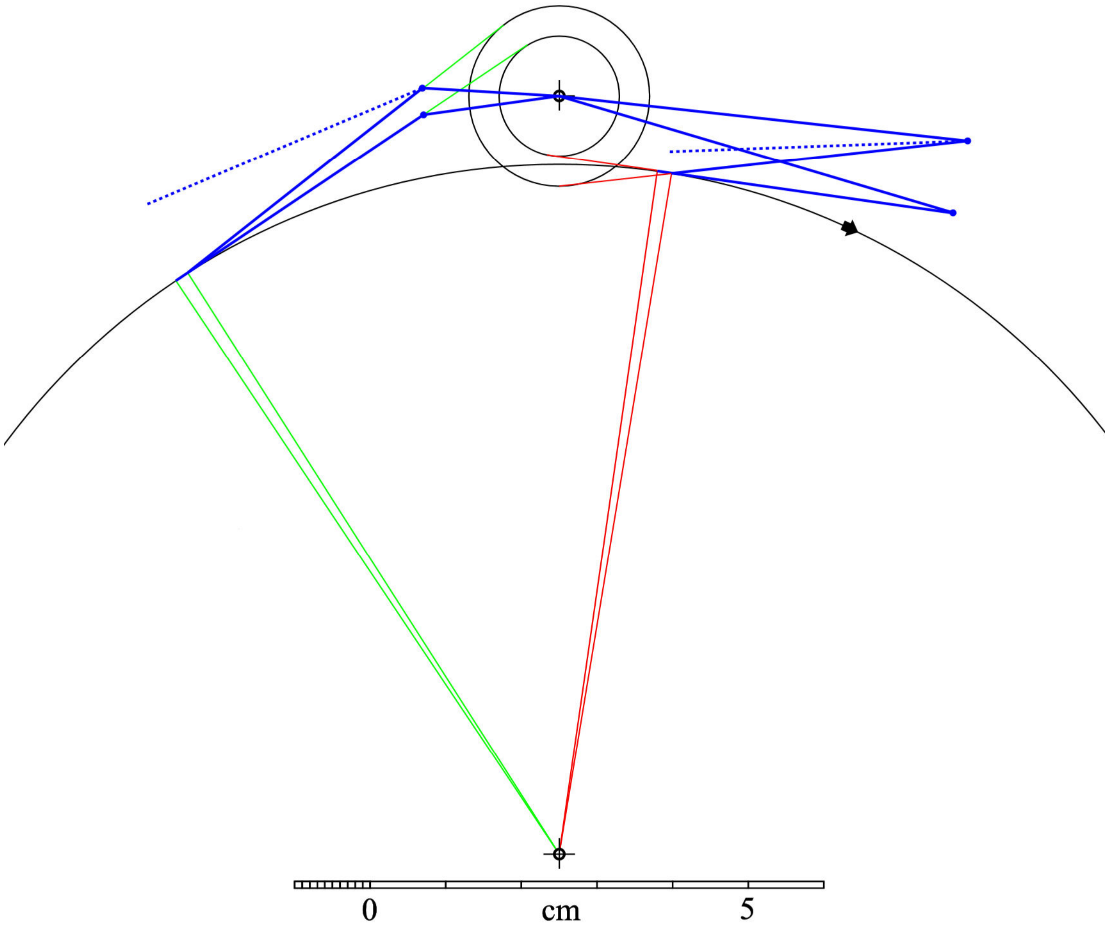
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 120
Escape wheel teeth tips pitch circle radius, R* mm = 74.75469
Mean number of teeth spanned, n = 15.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 9.34150
Entry pallet locking corner to pivot, JP mm = 34.96075
Exit pallet locking corner to pivot, DF mm = 34.96075
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm = 83.81952
Escapement frame arbor to entry pallet pivot, PZ mm = 17.57109
Escapement frame arbor to exit pallet pivot, FZ mm = 49.21312
Entry pallet locking corner lift upon release, J'K mm = 9.65671
Exit pallet locking corner lift upon release, CD' mm = 2.88399
Pendulum arc, p degrees = 11.12791

FIGURE 7.43 - 120 TOOTH ESCAPE WHEEL



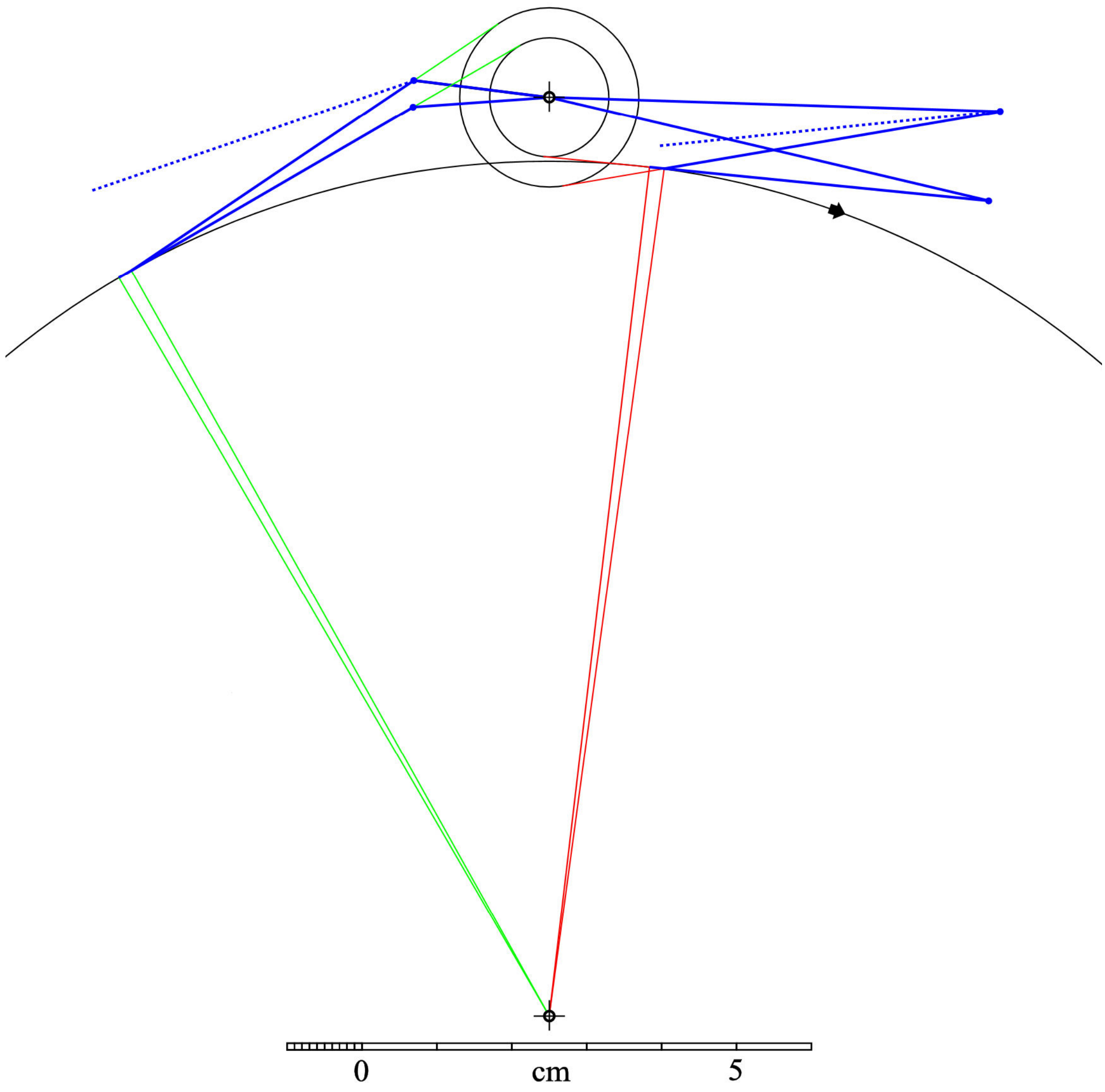
DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 150
Escape wheel teeth tips pitch circle radius, R* mm = 91.27208
Mean number of teeth spanned, n = 17.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 8.21050
Entry pallet locking corner to pivot, JP mm = 39.43152
Exit pallet locking corner to pivot, DF mm = 39.43152
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm = 100.25292
Escapement frame arbor to entry pallet pivot, PZ mm = 18.15791
Escapement frame arbor to exit pallet pivot, FZ mm = 54.33391
Entry pallet locking corner lift upon release, J'K mm = 10.38603
Exit pallet locking corner lift upon release, CD' mm = 2.93382
Pendulum arc, p degrees = 10.85492

FIGURE 7.44 - 150 TOOTH ESCAPE WHEEL



DESIGNER CHOSEN PARAMETERS

Total number of escape wheel teeth, N = 180
Escape wheel teeth tips pitch circle radius, R* mm = 113.77939
Mean number of teeth spanned, n = 18.5
Equivalent pendulum length, L* mm = 994.156
Angle 'd' degrees = 6.7
Entry pallet locking corner to pivot, JP mm = 45.43046
Exit pallet locking corner to pivot, DF mm = 45.43046
Mean torque arm ratio = 2/3
Mean torque arm, M* mm = 9.94156

CALCULATED PARAMETERS

Escape wheel to escapement frame arbor, OZ mm = 122.56971
Escapement frame arbor to entry pallet pivot, PZ mm = 18.23515
Escapement frame arbor to exit pallet pivot, FZ mm = 60.25793
Entry pallet locking corner lift upon release, J'K mm = 11.91687
Exit pallet locking corner lift upon release, CD' mm = 3.03688
Pendulum arc, p degrees = 11.22949

FIGURE 7.45 - 180 TOOTH ESCAPE WHEEL

OBSERVATIONS

Output from the mathematical model leads to interesting and useful observations.

For each of the derived geometries, the choice of escapement mean teeth spanned (n) is limited to a whole number of tooth spaces, plus half a tooth space. As a consequence of such a coarse controlling parameter, some features of the geometries will exhibit slight deviations from an absolutely smooth progression.

OBSERVATIONS
IMPULSE VARIATION

By intention, no account has been taken of the variation in force along each pallet arm during impulse. The resultant maximum torque ratio error is estimated to be + 3.9 % (30 tooth escape wheel) to + 1.8 % (180 tooth escape wheel).

OBSERVATIONS
ACHIEVING HARRISON'S 2 TO 3 MEAN TORQUE RATIO

Every one of the escapements within the chosen range could be induced to comply precisely with Harrison's stipulation that the mean torque ratio must be two to three.

For all studied escape wheel tooth counts, perfectly symmetrical impulse was achieved. In all cases, the entry torque ratio is two to three and the exit torque ratio is two to three. As a review of illustrations 7.40 to 7.45 will confirm, the additional incorporation of Stipulation 2 (the mean torque arm must be one hundredth of the equivalent pendulum length), results in pairs of torque arm circles that are identical throughout the entire range of presented geometries. Obviously, such a beautiful, matched series of geometries relies upon the assumption that the forces along the pallet arms are constant throughout the entire operating cycle.

OBSERVATIONS
ESCAPE WHEEL SIZE

The most obvious effect of increasing the escape wheel tooth count is that there is a consistent increase in the overall size of the escapement, most especially in the diameter of the escape wheel. Harrison's most refined movement, as fitted to his Final Regulator, was large by any standards (for sound reasons, beyond the scope of this publication), which enabled a large escape wheel diameter to be accommodated with ease. More conventional movements, of lesser size, might restrict the range of feasible escapements in some applications. By virtue of Stipulation 2 (i.e. the mean torque arm must be 1/100th of the equivalent pendulum length) the escapement and escape wheel sizes are inescapably dependent upon the chosen pendulum length, which, therefore, precludes the reduction of an overly large escapement and escape wheel in an attempt to match an unsuitable movement. Of course, a shorter pendulum would resolve such issues, were it not for Stipulation 5 (i.e. a 'long pendulum' shall be incorporated). All of Harrison's pendulums were seconds beating (or close to seconds beating) and that length should be regarded as the absolute minimum.

OBSERVATIONS
TORQUE TO THE ESCAPE WHEEL

In CSM, Harrison emphasises the importance of avoiding low driving torque to the escape wheel. Unavoidable variations in friction inevitably occur throughout any movement train and their cumulative effects can be surprisingly considerable. Most especially, any slight variations in arbor pivot and gearing friction towards the top of the train (i.e. closest to the escapement) will have a greater proportional effect upon smaller escape wheel driving torques than larger ones. In that respect, as large a diameter escape wheel as possible is to be preferred, since, for a given impulse at the escape wheel tooth tips, a larger diameter escape wheel will require a higher driving torque to the escape arbor than a smaller diameter escape wheel.

OBSERVATIONS
PALLET NIB LENGTH

The "length" of the exit pallet nib (in particular, the length of the lower of the two 'prongs', which meet to form the locking corner) must be chosen with care. As explained previously, the exit nib, if sufficiently long, provides some protection against escapement trip and escape wheel runaway should torque be applied to the escape wheel with the pendulum stationary and the exit pallet released. If the nib is too short, that protection will be lost. In addition a longer pallet nib will offer protection from escapement trip over a greater range of pendulum operating arc than a shorter nib. However, if the exit nib is too long, it will fail to clear the path of the escape wheel tooth tips after pallet release, thus preventing further escape wheel rotation. In an idealised world, an exit pallet nib of optimum length would barely clear the escape wheel after release. In the real world, other influences must be considered and are discussed elsewhere.

OBSERVATIONS
ESCAPE WHEEL TEETH CIRCULAR PITCH

The circular pitch of the escape wheel teeth (escape wheel tooth tips pitch circle circumference divided by the number of escape wheel teeth) is virtually constant across the presented range of optimised escapements. The chordal pitch (shortest distance between tooth tips) unsurprisingly displays a similar degree of regularity. Such regularity in the spacing of escape wheel teeth, virtually regardless of tooth count, is of considerable practical significance, discussed in other sections.

OBSERVATIONS
ESCAPE WHEEL TOOTH PROFILE

It is all too easy, whilst absorbed in the process of analysing pure geometries and imaginary escape wheel teeth, to overlook a requirement for adequate escape wheel tooth mechanical strength and resistance to impact damage. The leading edge of each tooth must be undercut, in order to ensure contact exclusively at the pallet locking corner and to provide clearance from the pallet nib during overswing. The trailing edge of the tooth form must remain clear of the released, descending pallet nib at all times. Those constraints define the shape and maximum size of an "envelope" within which the escape wheel tooth profile must fit. When such envelopes are arranged around the periphery of an escape wheel, their points of overlap will occur at the maximum depth of the spaces between adjacent escape wheel teeth. If that depth is insufficient to accommodate the required pallet nib length, then the escape wheel tooth form must be adjusted in order to provide a deeper gap. Whilst maintaining a constant tooth spacing, that can only be achieved by altering the orientation of the trailing edge. Such adjustment will weaken the tooth and, most significantly, produce a sharper, more delicate tip. This renders the tooth more susceptible to damage, which is especially undesirable in view of the tendency of the grasshopper escapement to trip and for the escape wheel to runaway at high speed.

OBSERVATIONS
ESCAPE WHEEL RECOIL

Recoil has already been identified as irrelevant, for all practical purposes, most especially when occurring in combination with Harrison's type of train, which accepts reversed rotation with virtually frictionless aplomb.

It is misleading to assume that the effects of recoil will reduce as escape wheel diameter increases. For a constant escape wheel tooth circular/chordal pitch irrespective of tooth count, it follows that the amount of recoil, in terms of movement at the tooth tips, will also be constant, to all intents and purposes. The angular rotation of the escape wheel and escape pinion during recoil will, therefore, reduce as escape wheel tooth count, and related diameter, increases. However, the remainder of the train will require adjustment in order to maintain a constant outcome at the centre arbor, which effectively subjects the upstream train to undiminished recoil, regardless of escape wheel tooth count.

OBSERVATIONS
PALLET LOCKING CORNER LIFT UPON RELEASE

Exit pallet locking corner lift, CD', varies from 2.32406 mm (30 tooth escape wheel) to 3.03688 mm (180 tooth escape wheel). Such moderate lifts and remarkably small variation across the range of geometries contrasts markedly with the behaviour of the single pivot grasshopper. This is an especially favourable characteristic of the twin pivot configuration, in that maximum exit pallet nib length for optimum trip protection may be easily accommodated within every escape wheel. Furthermore, escape wheel tooth strength and resistance to damage during runaway will be excellent in every case. Such exceptionally consistent characteristics greatly ease the task of the designer.

In contrast, apart from marked variation across the range of geometries, it is clear that entry pallet lift, J'K, is consistently much greater than exit pallet lift, CD', which renders the entry pallet almost useless in preventing escapement trip.

OBSERVATIONS
CHOICE OF ESCAPE WHEEL TOOTH COUNT

From previous comments, it must, by now, be clear that he most fundamental decision demanded of the grasshopper escapement designer is the number of escape wheel teeth to be incorporated. Many other factors are directly related to that single parameter. The range studied herein offers an adequate choice of 30, 60, 90, 120, 150 or 180 teeth, together with a multitude of observations regarding their characteristics.

In CSM, Harrison was absolutely clear that, for his single pivot escapement, a 120 teeth (4 mins.) escape wheel should be used, although he typically fails to explain his reasoning. His Final Regulator, which was presumably the most refined embodiment of his principles of precision timekeeping, incorporated just such an escapement. When selecting the optimum escape wheel tooth count for the twin pivot configuration, the observations listed herein do not all lead in the same direction, nor do they converge towards a single solution. A great deal depends upon the willingness of the designer to accept compromises.

Difficulties might arise as a consequence of the large escape wheel diameter required to accommodate high tooth counts. For example, the 120 tooth escape wheel, at almost 150 mm in diameter, might require careful incorporation into a conventionally sized movement. Solutions have already been discussed, but much depends upon the nature of each individual installation.

Of potential significance, unlike the single pivot escapement, the twin pivot configuration accommodates low tooth counts. This is an apparent advantage of the twin pivot escapement when space in the area of the escape wheel is at a premium. The most significant performance disadvantage of smaller escape wheels is that the required driving torque to the escape wheel arbor is reduced, when compared to wheels of greater tooth counts, thus rendering the escapement proportionally more susceptible to given variations in that torque. The twin pivot grasshopper is, however, considerably more demanding of space on the exit side (see ESCAPEMENT SIZE).

The 150 and 180 tooth escape wheel configurations are extremely large, by any standards. It could be argued that, if performance is the only consideration, such factors are irrelevant and that the greater required torques at the escape wheel arbor can only be an advantage.

OBSERVATIONS
ESCAPEMENT SIZE

The escape wheel is only one consideration in the process of deciding which geometry may be accommodated within a given space. The twin pivot escapement frame on the exit side is of considerable length and occupies a large swept area during normal operation. It is all too easy to forget, when studying pure geometries, that the exit pallet tail also extends beyond the escapement frame, which adds to the already difficult problem. Fortunately, in regulator movements, the area in question is often vacant. Much, therefore, depends upon each installation, for many are capable of absorbing this feature with ease.

OBSERVATIONS
MECHANICAL CONSIDERATIONS

The objective during nib capture is to present the nib locking corner to the applicable escape wheel tooth tip with absolute precision In that respect, the exit side of the twin pivot escapement raises potential problems, in terms of the necessary quality of manufacture and adjustment. There is a considerable distance between the escapement frame arbor and pallet arm pivot. Although the "effective" geometry merely consists of dimension Z to D (or C), it must be appreciated that the escapement frame arbor to exit pallet pivot and pallet pivot to nib locking corner dimensions must be carefully incorporated and that free play at the arbor or pivot must be minimal, in order to located D (or C) correctly. The setting of the exit composer might also demand considerable care, subject to the mechanical arrangement. Whilst such comments apply no less to any components within any grasshopper mechanism, difficulties may well arise in these particular areas. However, once successfully constructed, adjusted and proven, any well designed grasshopper escapement will offer centuries of consistent service, by virtue of there being, to all intents and purposes, no sliding friction or wear.The objective during nib capture is to present the nib locking corner to the applicable escape wheel tooth tip with absolute precision. If such precision is to be achieved, it is vital that all active dimensions be manufactured to the closest possible tolerances. All pivots and arbors must be free from 'shake' (free play), but must offer no resistance to rotation.

Pallet nib locking corners must be extremely sharp and precisely perpendicular to the plane of the escape wheel. Escape wheel tooth tips must also be sharp, although a *very slight* flat, tangential to the pitch circle, will reduce any tendency to deform, whilst improving resistance to damage should the escapement trip. The pitch circle diameter must not be reduced by formation of the slight flat, which will demand some thought and care when the teeth are initially formed. A rounded end, however slight, must be avoided, being insecure during the capture phase of operation, when the generation of adequate static friction is demanded.

The choice of materials is an important contributor to successful operation and longevity. Lignum vitae is a dense, naturally greasy wood, offering excellent wear resistance and low friction. It was used extensively by Harrison as a maintenance-free bearing material, with a potential life span measured in centuries. As such, it is an ideal bearing material at the pallet arms pivots. However, lignum vitae is not suitable as a pallet nib material, low friction being the complete opposite of what is required, demanding unnecessarily high escape wheel torque for reliable capture. Compromise hardwoods, such as hard oak, will offer useful service and will enable pallet arms to be manufactured as a single piece. Caution is required, however, in view of the tendency for oak to induce corrosion of certain materials. For example, the (non original) pallet arm balance weights of Harrison's Final Regulator, manufactured in lead, have required replacement due to severe corrosion. The resistance of chosen materials to damage during escape wheel runaway is also an important consideration, although damage to the pallet nibs is preferable to damage to the escape wheel.

Encouragingly, once correctly constructed, installed and adjusted, a grasshopper escapement will offer centuries of continuous, maintenance free operation, by virtue of there being, to all intents and purposes, absolutely no sliding friction or wear.

ELIMINATING ERRORS

The following sections will study all potential causes of error relating to the twin pivot grasshopper escapement.

Grasshopper escapement errors are, in most cases, markedly different to those of the anchor or dead beat escapements. In fact, in most cases, errors are non-existent, as will be explained. Nevertheless, it will be instructive to repeat the topic headings presented for the anchor and dead beat escapements and provide comments applicable to the twin pivot grasshopper escapement, in order to demonstrate the numerous advantages.

ELIMINATING ERRORS
RECOIL

The twin pivot grasshopper escapement is a recoil escapement. By virtue of the large pendulum amplitude and energy, recoil can be quite marked. Nevertheless, there are no adverse effects on the escapement itself. There is no sliding friction, no wear and, therefore, no increase in wear due to the high loads often generated by recoil.

The effects of grasshopper recoil upon a conventional train are no better than those produced by the anchor and are, clearly, worse than those of the dead beat. Solutions include the abandonment of the common form of train and the adoption of a Harrison train, which functions with virtually no sliding friction, no lubrication and an extremely low resistance to reverse rotation. Another extremely useful option is the fitment of an escape arbor remontoire. In simplified terms, a remontoire isolates the escape wheel (and, therefore, the escapement and pendulum) from the movement train. Torque delivery to the escape arbor is rendered either extremely constant, or cyclically regular and recoil of the escape wheel is absorbed by the remontoire, rather than transmitted to the train.

ELIMINATING ERRORS
CIRCULAR ERROR

In complete contrast to the philosophies of the anchor and dead beat escapements, Harrison specifically demands that the amplitude of the pendulum must be large. One feature of the grasshopper escapement is that it is capable of generating such large arcs with no adverse effects. Harrison specifies a maximum arc of fifteen degrees.

What Harrison has done is typically confident and bold, born of establishing honest truths and adopting straightforward thinking. He is aware that undesirable disturbances to the motion of a pendulum will have a proportionally greater effect on regular motion if the pendulum is possessed of low energy, rather than high energy. He therefore advocates a pendulum with high energy, which he achieves by incorporating an escapement geometry producing a large amplitude. He is thereby consciously encouraging considerable circular error, which he then tames by fitting effective pendulum suspension cheeks, to his own, unique specifications.

Harrison determined that cycloidal cheeks to either side of the pendulum suspension spring, in combination with the characteristics of his early grasshopper escapement, tended to provide unsatisfactory compensation. He determined, no doubt by careful experimentation, that slightly more "open" cycloidal cheeks seemed to be required instead. When he introduced his stipulation that the mean torque ratio should be 2 to 3, he would appear to have discovered that the cycloid, whether "open" or not, was no longer valid and that cheeks in the form of a remarkably simple circular arc were appropriate. The degree of compensation could be refined by altering the radius of the circular arc, which was a much easier task than the modification of a cycloidal form. It can only be concluded that the 2 to 3 torque ratio is modifying circular error to a sufficient extent that modified pendulum suspension compensation is required.

ELIMINATING ERRORS
ERRORS DUE TO WEAR

The twin pivot grasshopper escapement operates without any sliding friction whatsoever, apart from an insignificant rotation at the pallets pivots. Based upon the longevity of the Brocklesby escapement pallets pivots, they would appear to have a working life measured in centuries, provided that materials are chosen sensibly.

Static friction at the pallet nib locking corners cannot produce wear, although impact between those corners and the escape wheel tooth tips often produces an insignificant indentation on new wooden pallets, which stabilises once formed and may be ignored.

Put simply, there is, effectively, no wear. There cannot be, therefore, any errors due to wear.

ELIMINATING ERRORS
LUBRICANT ERROR

For escapement configurations such as the anchor and dead beat, it is necessary to apply lubricant in order to reduce friction and wear to acceptable levels. Unfortunately, clock oils, be they of animal, mineral, vegetable or synthetic origin, are subject to evaporation, oxidation, contamination, molecular modification etc. leading to a degradation in their wear reducing properties. Of considerable significance to the generation of errors, such lubricant deterioration will alter the magnitudes of viscous drag and frictional forces.

In complete contrast, the twin pivot grasshopper escapement generates no sliding friction. There is, therefore, no requirement for lubrication at any point of the escapement, thereby completely eliminating that source of errors.

In addition, Harrison's movements were, by ingenious design, entirely free from sliding friction, apart from the insignificant movements of self lubricating lignum vitae rollers and hubs upon fine brass axles. Lubrication and any errors due to lubrication are thereby entirely eliminated.

ELIMINATING ERRORS
ESCAPEMENT ERROR

The most ideal timekeeping (ignoring the use of evacuated, temperature regulated enclosures) will be achieved by a completely free swinging pendulum. An effect associated with counting pendulum swings and adding energy by mechanical means via an escapement is that, no matter how delicately we count or how carefully we add energy, we will inevitably interfere with the otherwise free swing of the pendulum, altering its period. Any alteration in the period of the pendulum caused by the escapement is called escapement error. For common escapements, such as the anchor and dead beat, the degree of escapement error will depend upon how constant the torque supplied to the escape wheel is, how the impulse from the escape wheel to the pallets varies, how friction alters during each cycle and over many cycles, how consistent the applied lubricants are, how much of pendulum arc the impulse, drop and recoil events occupy and at what positions of the pendulum arc they occur.

Studies of escapement error for the anchor and dead beat escapements identify lubricant degradation in combination with drop as the primary causes of variations in escapement error. The twin pivot grasshopper escapement completely eliminates those sources of error, by virtue of not only operating without lubrication, but also, astonishingly, without drop. The escape wheel is never free from the escapement, since one escape wheel tooth is always engaged with a pallet locking corner and pallet capture and release events are simultaneous. There can, therefore, be absolutely no variations in escapement error due to lubricant or drop. Thus, although continuous pallet engagement with the escape wheel is as far removed from the ideal of a completely free pendulum as it could possibly be, it achieves considerably better performance than escapements incorporating phases of freedom.

ELIMINATING ERRORS
CAUSES OF ESCAPEMENT ERROR

This section will consider the various causes of escapement error, taking particular care to note that many other sources of error not relevant to the escapement will not be considered, unless stated otherwise.

(A) - Torque supplied to the escape wheel.

The driving weight supplies energy to the movement train, which emerges as a torque about the escape arbor and thence as a force at the engaged escape wheel tooth tip.

Harrison's movements, as already explained, required no lubrication. His virtually frictionless gearing, however, generated marked variations in torque at the escape arbor. Harrison eliminated those variations by fitting his own, typically ingenious, frictionless, high performance, remontoire, rendering torque to the escape arbor virtually constant.

(B) - Torque transmitted by the pallets.

The transmission of escape arbor torque to the escapement frame arbor, the crutch and the pendulum occurs without modification by friction, for there is no friction. Apart from slight 'impact' pitting to the locking corners of new pallets, which rapidly stabilises, there is no wear to the escapement or escape wheel. A a consequence, the twin pivot grasshopper escapement exhibits stable torque transmission for many centuries of operation.

(C) - Effect of variations in drop.

Studies of escapement error for the anchor and dead beat escapements have clearly identified and explained how lubricant degradation in combination with escape wheel drop is a significant cause of unpredictable variations in escapement error. The twin pivot grasshopper escapement completely eliminates those sources of error, by virtue of not only operating without lubricant, but also, astonishingly, entirely without drop. The escape wheel is never free from the escapement, since one escape wheel tooth is always engaged with a pallet locking corner and pallet capture and release events are, effectively, simultaneous. If there is no lubrication or drop, then there can be absolutely no escapement error due to lubrication or drop and, therefore, no associated variations in escapement error. As a result, although continuous pallet engagement with the escape wheel is as far removed from the ideal of a completely free pendulum as it could possibly be, it achieves far superior performance, when compared to escapements incorporating phases of freedom.

(D) - Effect of variations in pendulum amplitude.

Variations in pendulum amplitude will be absorbed by the escapement in the form of variations in escape wheel recoil, for no other phase of the operating cycle can vary.

The grasshopper escapement incorporates recoil without generating errors. Recoil commences at the point of pallet nib locking corner capture and advances no further when the motion of the pendulum ceases at the extremity of is motion. Variations in the extremity of swing of the pendulum are of no consequence, except for their effect upon circular error.

Harrison employed large pendulum amplitudes and circular suspension cheeks, in order to encourage circular error, which he then tamed. Adjustment of the radii of the circular cheeks renders the pendulum isochronous, (constant period, regardless of amplitude, within sensible limits). Thus, variations in pendulum amplitude have, in theory at least, no effect whatsoever. In practice, much depends upon the care taken in adjusting the cheek radii.

DEFICIENCIES

Having described the many advantages of the twin pivot grasshopper escapement, we should also, in fairness, summarise its deficiencies.

By far the greatest problem is the possibility of trip, with the associated potential for escape wheel high speed runaway and damage. It is, in truth, inaccurate and unfair to describe this as a tendency, for if the escapement is correctly constructed, adjusted and operated, there are no valid reasons for trip to occur. Perhaps it could be more accurately described as an escapement that will, not unreasonably, 'bite back' more severely than conventional escapements, if mistreated by an ignorant or clumsy operative.

A potential source of difficulty is created by the length of the escapement frame required to support the exit pallet pivot (see MECHANICAL CONSIDERATIONS). All dimensions and pivots must be of the highest accuracy and quality, if problems are to be avoided.

The construction and adjustment of a grasshopper escapement is slightly more involved than, for example, the anchor or dead beat, although it could hardly be described as a major undertaking. As a proportion of the work involved in creating an entire precision regulator, the construction of a grasshopper escapement is a relatively minor task. It must be bourne in mind that the anchor and dead beat escapements are subject to wear, require regular cleaning and lubrication and will eventually demand repair, whereas the grasshopper escapement, once constructed and adjusted, is capable of operating without any attention for many centuries.

The only other difficulty, whether regarded as aesthetic or practical, arises from the large pendulum amplitude, which, at first sight, will require an unusually wide case. For his early regulators, Harrison incorporated (possibly retrospectively) hollow "cheeks" to the case sides at pendulum bob level, in order to accommodate the pendulum in full swing. This solution permits the use of a case of otherwise conventional width and appearance.

A subtle deficiency arises from Harrison's universal use of wood in the construction of the pallet arms. In typical clockwork (and including Harrison's Final Regulator), the escapement frame, main movement plates, escape wheel and crutch would be made of brass. The composers of the grasshopper are commonly of brass. Differential thermal expansion of wood and brass components would certainly alter the geometry and behaviour of the escapement. There would, in addition, be dimensional and shape alterations to the pallet arms as the wood of which they were made reacted to changes in atmospheric temperature and humidity. Such undesirable effects could be eliminated by constructing the pallet arms of brass, suitably lightweight in form. The nibs should still be of a suitable wood, such as oak and the pivot bearing would, ideally, be of lignum vitae.

CONCLUSIONS

The twin pivot configuration has been shown to be extremely versatile. A wide range of escape wheel sizes and tooth counts have been accommodated without difficulty and any sensible torque ratio may be incorporated.

The twin pivot escapement is remarkably free from inconsistent behaviour. There is no wear, no requirement for lubrication, no errors due to lubricant deterioration, no drop or associated errors and no marked variation in delivered torque. There is a requirement for pendulum suspension cheeks, the success of which depends entirely upon accurate manufacture and adjustment. Performance must, inevitably, exceed that of the anchor and dead beat escapements by a considerable margin.

Maintenance is negligible, being confined to occasional cleaning.

Methods have been presented enabling the creation of twin pivot grasshopper geometries complying with any sensible demands. A group of optimised geometries successfully incorporating Harrison's stipulations have been produced and presented. Studies of escape wheel tooth counts from as low as thirty to as high as one hundred and eighty have revealed no geometrical or practical obstacles.

Exit pallet locking corner lift is quite consistent across the chosen range and offers excellent trip protection in combination with good escape wheel tooth strength and resistance to damage, most especially of the tips.

A mathematical design tool has been devised and proven. It is capable of creating any number of twin pivot grasshopper escapement geometries in accordance with any realistic stipulations.

It may be that subsequent research, using the design tools provided, will expose flaws in Harrison's stipulations and identify improvements or perfection in others. The twin pivot configuration defined by the analysis presented herein should be capable of modification to suit any such revelations.

A potentially significant consequence of the development of a viable mathematical modelling technique may prove to be the incorporation of absolutely symmetrical torque ratios, in terms of a contribution to timekeeping performance..

The above conclusions leave the most vital question of all unanswered:

Why did John Harrison apparently abandon the twin pivot grasshopper escapement, in favour of his single pivot configuration?

This study, most especially when compared to a separate work dealing with the single pivot grasshopper (see BIBLIOGRAPHY), has demonstrated that the twin pivot configuration is superior in every respect.

Has this analysis failed to recognise an explanation for Harrison's apparent preference?

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